Thin-Walled Structures 73 (2013) 101–111

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Analytical and finite element modelling of long plate mode jumping behaviour

H. Hofmeyer *, J. Courage

Technische Universiteit Eindhoven, Department of the Built Environment, Structural Design Group, VRT 9.32, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

ARTICLE INFO

Article history:
Received 20 December 2012
Received in revised form
24 June 2013
Accepted 25 June 2013
Available online 23 August 2013

Keywords:
Mode Jumping
Analytical Modelling
Finite Element Modelling

ABSTRACT

Trapezoidal sheeting of thin-walled steel is applied frequently for roofing and cladding. As such, it is loaded by a concentrated load (at the support) and a bending moment. A recently developed model to predict the sheeting's failure behaviour leaves the question open whether mode jumping (the phenomenon where a plate dynamically changes its buckling mode during an increasing load) should be taken into account in the model. This article presents the analytical and finite element modelling of square and long plates, which, depending on the boundary conditions, may represent the compressed flange of trapezoidal sheeting. The analytical modelling is based on the combination of several displacement functions and using the principle of minimal potential energy. Hereafter the stability of each part of the resulting equilibrium curves is determined. A spin-off of the analytical model is an analytical expression for a current curve-fitted based prediction formula for the post/pre-buckling stiffness ratio by Rhodes. Furthermore, the accuracy range of a solution by Williams and Walker for the far-post buckling behaviour can be confirmed. The finite element modelling has been carried out by implicit dynamic, and explicit (dynamic) simulations. Both for the load levels and the buckling mode sequences, the analytical and finite element models give equivalent results. It is concluded that for the specific boundary conditions that represent the situation of a compressed flange for trapezoidal sheeting, it is very likely that mode jumping will not occur.

1. Introduction

Sheeting of thin-walled steel is widely used for the construction of building roofs and claddings. Near the intermediate supports, which are either hot-rolled steel beams or cold-formed sections, the sheeting is loaded by a combination of bending moment due to distributed loads on the sheeting and a concentrated load due to the support reaction, Fig. 1a.

Either mainly the concentrated load leads to failure via so-called web-crippling [1–5], or the combination of web-crippling and bending moment may induce failure [6–12]. Current design codes [13,14] predict failure due to this latter load combination as follows. First, the ultimate bending moment M_u that the sheeting can withstand is predicted without taking the action of the concentrated load F into account. This prediction is carried out by the semi-analytical effective width method which is based on the work of Von Kármán [15] for very slender plates and the additional work of Winter [16] for plates with a slenderness normally used in sections of cold-formed steel. The North American and Australian specifications also allow the use of the direct strength method [17,18] as an alternative to the effective width method. For this method, not the separate compressed elements, but the whole cross-section is analysed for local, global, and distortional buckling, resulting in the critical bending moment. The ultimate bending moment can then be found by curve-fitting the relationship between critical moment, normalised with the moment of first yield, and the ultimate bending moment, also normalised. Secondly, the design codes require the prediction of the ultimate concentrated load R_w, defined as web-crippling load, without taking the effect of the bending moment M into account. This prediction of R_w is purely based on curve-fitting of test-results [19–38]. Finally, sheeting failure by a combination of bending moment and concentrated load is predicted by an interaction rule, as shown in Fig. 1b, which is also purely based on curve-fitting of test-results [19,21,39–42]. The ultimate corresponding reaction load on the section is defined as load F_u.

Fig. 1b clearly shows that the procedure in the current design codes is a combination of several procedures, both semi-analytical and curve-fitting, and it is not based directly on the occurring failure modes. Therefore, the ultimate failure model was developed [43], conceptually presented in Fig. 2 showing a part of the compressed flange.

* Tel.: +31 40 247 2203.
E-mail address: h.hofmeyer@bwk.tue.nl (H. Hofmeyer).

0263-8231/$ - see front matter
http://dx.doi.org/10.1016/j.tws.2013.06.017

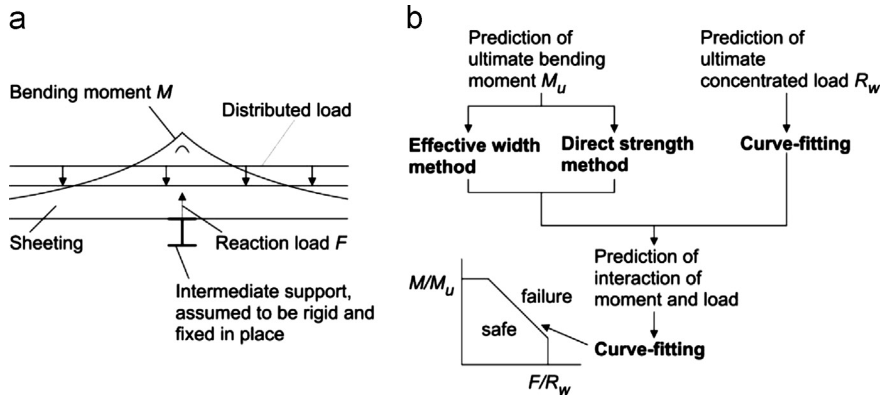

Fig. 1. (a) Sheeting loaded by combination of moment and force and (b) current design codes.

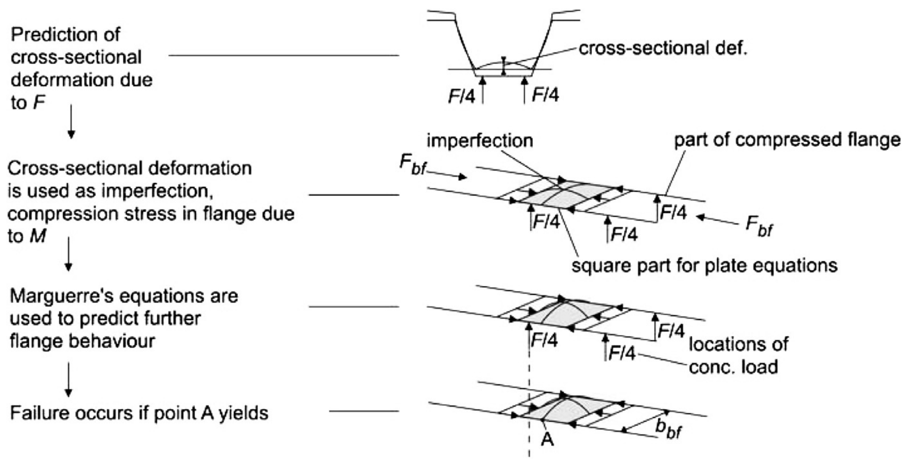

Fig. 2. Concept of ultimate failure model.

Experiments [43] showed that the elastic deformation of the compressed flange was always such that the relevant part of it could be modelled by a square element, shown shaded in Fig. 2, starting at a distance of 0.25 times the flange width to the concentrated load application. The concentrated load F on the sheeting is modelled as four point loads $F/4$. Due to these four point loads, the cross-section deforms and as a result the compressed flange shows an out-of-plane deflection. Besides this effect, the concentrated load F also causes a bending moment M on the sheeting. This bending moment causes a compression force F_{bf} in the flange that can be made equivalent to a certain average compression stress in the flange. Now, Marguerre's [44] equations are used to determine the (initial post-buckling) plate behaviour for the initial imperfection (that is the out-of-plane deflection caused by the concentrated load) and the compressive stress caused by the bending moment. As soon as the outer-fibre stress in point A is such that the yielding stress is reached, the sheeting is assumed to fail. Note that the action of the concentrated load is taken into account only indirectly, via the cross-sectional deformation and the compression stresses due to the bending moment, and that local additional stresses at A directly due to the concentrated load are not incorporated.

This ultimate failure model performs well compared to current design codes; however, the plate equations of Marguerre have three disadvantages. First of all they are too tedious to use as a design rule. Secondly, they are only able to predict failure by first yield of a single location in the plate. However, research has shown that the failure of a compressed plate can occur in two different modes and for these modes, first yield is not always a good predictor of the ultimate load [45]. Finally, they are only valid for the initial post-buckling behaviour (loads up to about two times the buckling load) in which it is assumed that the displacement fields before and after buckling are the same. Therefore, the fictitious strain method has been developed, which replaces Marguerre's equations, is easier to use, is also suitable for far post-buckling behaviour, and provides additional insight into the plate failure behaviour [45]. The fictitious strain method has already been used and discussed in literature [46–50].

The ultimate failure model, preferably used in combination with the fictitious strain method instead of Marguerre's equations, is not suitable for second-generation sheeting, which is sheeting with longitudinal stiffeners. This is the case because the fictitious strain method (and also Marguerre's equations) had been developed for first-generation sheeting, which is sheeting with flat plates only. Therefore research was started on the application of the fictitious strain method on second-generation sheeting, thus on plates with longitudinal stiffeners [51]. Using the finite element method for this research, it came out that plates with longitudinal stiffeners could only be modelled as long plates, instead of the computationally more efficient square plates as used for flat plates. But even before these plates with longitudinal stiffeners were simulated, instabilities occurred in the finite element analyses for the flat long plates and it was thought that among these instabilities also mode-jumping phenomena existed [51]. This was regarded as a serious drawback as mode jumping phenomena would undermine the principles of the fictitious strain method. Therefore a new research project was initiated, presented here, in which the mode jumping phenomena was investigated in more detail. Besides a new analytical approach to model mode jumping behaviour analytically, the finite element models used indicated