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Buckling behaviour of thin-walled regular polygonal tubes subjected to bending or torsion

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ABSTRACT

In a recent paper (Gonçalves and Camotim, 2013 [1]), the authors presented an investigation concerning the buckling (bifurcation) behaviour of uniformly compressed thin-walled tubes with regular polygonal cross-sections (RCPS). The present paper complements the previous work by addressing the local and distortional buckling behaviour of RCPS members subjected to bending or torsion and aims at providing a novel insight into these phenomena. In particular, the specialization of Generalized Beam Theory (GBT) for RCPS, as recently proposed in Gonçalves and Camotim (2013) [2], is employed to obtain closed-form analytical solutions and also to carry out parametric studies by means of numerical analyses which are both computationally efficient (due to the small number of d.o.f. involved) and clarifying (due to the modal decomposition features of GBT). For validation purposes, solutions taken from the literature and also standard shell finite element model results are employed.

1. Introduction

The widespread popularity of thin-walled tubular structural members with regular (equiangular and equilateral) convex polygonal cross-sections (RCPS) is well-known (e.g., [3–5]). For instance, the applications of these members in the construction industry range from communication towers, lighting posts and transmission line structures, to railroad sign frames and railway overhead contact line posts. In spite of this popularity, it is also recognized that there is limited information on the buckling (bifurcation) behaviour of these members [6].

In two recent papers [1,2], the authors investigated the general mechanics of deformation of thin-walled RCPS tubular members and also their buckling behaviour under uniform compression. These investigations were based on a specialization of Generalized Beam Theory (GBT[1]) for RCPS, which takes advantage of the cross-section rotational symmetry in order to uncouple the equilibrium equations. This uncoupling made it possible to develop analytical solutions and also obtain accurate numerical solutions with just a few cross-section deformation modes. In particular, it was demonstrated that duplicate solutions are obtained in many situations.

This paper complements the previous work by addressing the local and distortional buckling behaviour of RCPS members under bending/torsion. As already mentioned, information on these topics is still relatively scarce. The local buckling of RCPS members under uniform bending has been investigated in [6], on the basis of results obtained with finite strip analyses. In this study, RCPS with 4–8 walls were studied and a 25% increase in buckling stress with respect to uniform compression was reported. The local buckling of long polygonal tubes in combined compression and torsion was examined by Wittrick and Curzon [10], using stability functions. For equilateral triangular sections, it was concluded that the torsional buckling stress is 3.7% higher than that for a simply supported plate under pure shear, whereas for square sections it is almost identical.

The outline of the paper is as follows. Section 2 briefly summarizes the fundamentals of GBT linear stability analyses for RCPS members subjected to uniform bending and torsion. Section 3 addresses the buckling behaviour of members under uniform bending. Local buckling in a "plate-type" mode (without displacements of the wall junctions), as well as in more general modes (with displacements of the junctions), is examined in detail. Section 4 focuses on buckling under torsion. Besides local plate-type buckling, pure distortional and mixed-mode local-distortional buckling are investigated. Finally, Section 5 presents the concluding remarks.

Throughout the paper, several closed-form analytical solutions are developed and the results of several parametric studies are presented and discussed. For validation purposes, the GBT-based

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[1] GBT is a beam theory that allows for general cross-section deformation (warping, distortion, etc.) through the consideration of the so-called "cross-section deformation modes". For a complete account of its origins and recent developments see, e.g., [7–9] and the references contained therein.

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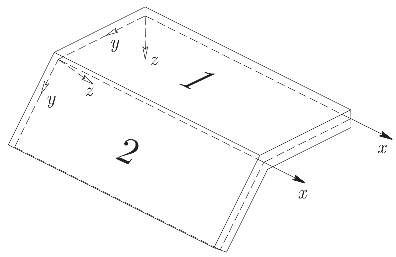

Fig. 1. Mid-surface local coordinate systems and associated displacement components of an arbitrary thin-walled member.

results are compared with solutions taken from the literature and also values obtained with shell finite element models, using ADINA [11].

The notation of the previous papers [1,2] is followed. Vectors and matrices are represented in **bold letters**. Partial derivatives are indicated by subscripts following a comma, e.g., $f_{,x}=\partial f/\partial x$. The δ denotes a virtual variation. The cross-section geometric and GBT discretization parameters (n, m) are as indicated in Fig. 2(a) and the non-dimensional parameters

$$\beta_1=\frac{L}{r}, \qquad \beta_2=\frac{r}{t}, \tag{1}$$

are employed, where L is the member length. Finally, the classical plate buckling formula is given by the following alternative expressions:

$$\frac{\sigma_{cr}}{E}=\frac{k\pi^2}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2=\frac{k\pi^2}{48(1-\nu^2)\beta_2^2\,\sin^2(\pi/n)}, \tag{2}$$

where E is Young's modulus, ν is Poisson's ratio and k is the buckling coefficient.

2. GBT linear stability analysis for RCPS

The GBT notation employed in [12] is adopted, with the local axes (x,y,z) for each wall indicated in Fig. 1. The displacement field is written as

$$\boldsymbol{U}(x,y,z)=\begin{bmatrix}U_x\\U_y\\U_z\end{bmatrix}=\begin{bmatrix}u(x,y)-zw_{,x}(x,y)\\v(x,y)-zw_{,y}(x,y)\\w(x,y)\end{bmatrix}, \tag{3}$$

$$u(x,y)=\sum_{k=1}^{D}\overline{u}_k(y)\ \phi_{k,x}(x), \tag{4}$$

$$v(x,y)=\sum_{k=1}^{D}\overline{v}_k(y)\ \phi_k(x), \tag{5}$$

$$w(x,y)=\sum_{k=1}^{D}\overline{w}_k(y)\ \phi_k(x), \tag{6}$$

where $\overline{u}_k$, $\overline{v}_k$, $\overline{w}_k$ are the components of the $k=1,\dots,D$ deformation modes and $\phi_k(x)$ are the associated amplitude functions, which constitute the problem unknowns.

For members subjected to pre-buckling uniform longitudinal normal stresses $\sigma_{xx}=\lambda\overline{\sigma}(y)$ and shear stresses $\sigma_{xy}=\lambda\overline{\tau}(y)$, where λ is the loading parameter and $\overline{\sigma}(y)$, $\overline{\tau}(y)$ are the reference stress distributions, the linearization of the virtual work equation yields the bifurcation equation system [13]:

$$\int_L\begin{bmatrix}\delta\boldsymbol{\phi}\\\delta\boldsymbol{\phi}_{,x}\\\delta\boldsymbol{\phi}_{,xx}\end{bmatrix}^t\begin{bmatrix}\mathbf{B}&\mathbf{0}&\mathbf{D}_2\\\mathbf{0}&\mathbf{D}_1&\mathbf{0}\\(\mathbf{D}_2)^t&\mathbf{0}&\mathbf{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{\phi}\\\boldsymbol{\phi}_{,x}\\\boldsymbol{\phi}_{,xx}\end{bmatrix}dx$$

$$+\lambda\int_L\begin{bmatrix}\delta\boldsymbol{\phi}\\\delta\boldsymbol{\phi}_{,x}\\\delta\boldsymbol{\phi}_{,xx}\end{bmatrix}^t\begin{bmatrix}\mathbf{0}&\mathbf{X}_\tau^{\overline{v}+\overline{w}}&\mathbf{0}\\(\mathbf{X}_\tau^{\overline{v}+\overline{w}})^t&\mathbf{X}_\sigma^{\overline{v}+\overline{w}}&\mathbf{X}_\tau^{\overline{u}}\\\mathbf{0}&(\mathbf{X}_\tau^{\overline{u}})^t&\mathbf{X}_\sigma^{\overline{u}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\phi}\\\boldsymbol{\phi}_{,x}\\\boldsymbol{\phi}_{,xx}\end{bmatrix}dx=0, \tag{7}$$

where $\boldsymbol{\phi}$ is a vector containing the amplitude functions $\phi_k(x)$ and, assuming wall transverse inextensibility ($\varepsilon_{yy}^M=0$), the GBT modal matrices are subdivided into the linear stiffness matrices:

$$\mathsf{B}_{ij}=\mathsf{B}_{ij}^B=\int_S\frac{Et^3}{12(1-\nu^2)}\overline{w}_{i,yy}\overline{w}_{j,yy}\,dy, \tag{8}$$

$$\mathsf{C}_{ij}=\mathsf{C}_{ij}^M+\mathsf{C}_{ij}^B=\int_S\left(Et\overline{u}_i\overline{u}_j+\frac{Et^3}{12(1-\nu^2)}\overline{w}_i\overline{w}_j\right)dy, \tag{9}$$

$$\begin{aligned}(\mathsf{D}_1)_{ij}&=(\mathsf{D}_1^M)_{ij}+(\mathsf{D}_1^B)_{ij}\\&=\int_S G\left(t(\overline{u}_{i,y}+\overline{v}_i)(\overline{u}_{j,y}+\overline{v}_j)+\frac{t^3}{3}\overline{w}_{i,y}\overline{w}_{j,y}\right)dy,\end{aligned} \tag{10}$$

$$(\mathsf{D}_2)_{ij}=(\mathsf{D}_2^B)_{ij}=\int_S\frac{\nu Et^3}{12(1-\nu^2)}\overline{w}_{i,yy}\overline{w}_j\,dy, \tag{11}$$

and the geometric matrices:

$$(\mathsf{X}_\sigma^{\overline{u}})_{ij}=\int_S\overline{\sigma}t\overline{u}_i\overline{u}_j\,dy, \tag{12}$$

$$(\mathsf{X}_\sigma^{\overline{v}+\overline{w}})_{ij}=\int_S\overline{\sigma}t(\overline{v}_i\overline{v}_j+\overline{w}_i\overline{w}_j)\,dy, \tag{13}$$

$$(\mathsf{X}_\tau^{\overline{u}})_{ij}=\int_S\overline{\tau}t\overline{u}_{i,y}\overline{u}_j\,dy, \tag{14}$$

$$(\mathsf{X}_\tau^{\overline{v}+\overline{w}})_{ij}=\int_S\overline{\tau}t(\underline{\overline{v}_{i,y}\overline{v}_j}+\overline{w}_{i,y}\overline{w}_j)\,dy, \tag{15}$$

where S denotes the cross-section mid-line, superscripts M and B designate membrane and bending terms, respectively, G is the shear modulus, $i,j=1,\dots,D$ and the underlined term in $\mathbf{X}_\tau^{\overline{v}+\overline{w}}$ vanishes due to the $\varepsilon_{yy}^M=0$ assumption, but is included for completion.

The cross-section deformation modes for RCPS may be determined from the classic procedure [7] or the specific procedure proposed in [2], which employs a discretization with n natural nodes and m equally spaced intermediate nodes in each wall, as shown in Fig. 2(a). With the specific procedure for RCPS, for each natural deformation mode set,[2] the GBT matrices are diagonalized using a coordinate transformation that employs vectors $\boldsymbol{a}^{(l)}$ and $\boldsymbol{b}^{(l)}$, with components $j=1,\dots,n$ given by

$$a_j^{(l)}=\cos\left(\frac{2\pi jl}{n}\right), \quad l=0,\dots,\left[\frac{n}{2}\right], \tag{16}$$

$$b_j^{(l)}=\sin\left(\frac{2\pi jl}{n}\right), \quad l=1,\dots,\left[\frac{n-1}{2}\right], \tag{17}$$

where $[k]$ denotes the largest integer not exceeding k. For each $l=1,\dots,[(n-1)/2]$, the associated vector pair ($\boldsymbol{a}^{(l)}$, $\boldsymbol{b}^{(l)}$) generates a duplicate GBT matrix diagonal component. For the particular case of the local $\overline{w}$ mode set, indispensable in local buckling analyses, it is possible to block diagonalize the GBT matrices and each vector pair ($\boldsymbol{a}^{(l)}$, $\boldsymbol{b}^{(l)}$) generates a particular $2m\times 2m$ diagonal block with m duplicate eigenvalues. In this paper, l is always employed to designate the vectors $\boldsymbol{a}^{(l)}$ and $\boldsymbol{b}^{(l)}$.

[2] The natural deformation mode sets are initially generated by imposing displacements at the cross-section natural nodes, e.g., the natural Vlasov warping and natural shear mode sets.