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Modal interactions of a geometrically nonlinear sandwich beam with transversely compressible core

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ABSTRACT

Modal interactions of a geometrically nonlinear sandwich beam with transversely compressible core in the presence of combination internal resonance are investigated. At first, a geometrically nonlinear, {2,1}-order theory is used to derived the equations of motion and the compatible boundary conditions of the beam. Then, Galerkin's weighted residual method and the multiscale approach are used to address the governing system. Next, modal interactions in the presence of combination internal resonance and subjected to primary-resonance excitation are investigated. Finally, the commercial code ABAQUS is used to validate the theoretical results we have obtained.

1. Introduction

A sandwich panel is a layered structure consisting of two thin face sheets which are bonded to a thick core layer. Within the principle of sandwich construction, the face sheets carry the tangential and bending loads while the core transmits the transverse normal and shear loads. This special configuration brings us an extremely low weight while retaining high bending stiffness thanks to the adopted lightweight low-density core. In view of its low weight, sandwich panels have extensive application in many fields of engineering such as aerospace, aeronautics, automotive, naval construction and civil engineering (see e.g., [1,2]). When the thick core layer is made of a weak material, sandwich panels would have complicated deformations, buckling behavior, and rich dynamic features [3]. For example, different from the standard laminated structures, the buckling modes of a sandwich panel with weak core can be present in two scales: (1) global or overall buckling, which is similar to Euler's buckling for homogeneous columns, and (2) some local forms of buckling of the face sheets called wrinkling (see e.g., [2,4] and the references thereof). Expectantly, similar phenomena would arise in the vibration responses of the sandwich panels.

It is well-known that nonlinear structures may display manifold nonlinear characteristics such as multiple solutions, limit cycles, subharmonic and superharmonic resonances, various modal interactions, bifurcations and chaotic motions (see e.g., [5, pp. xv–xviii]). Among these behaviors, modal interactions pose a particular concern since interactions between global and local modes of a sandwich panel due to its soft and thick cores can cause disastrous results if vibrational energy is transferred from low-amplitude high-frequency modes to lower modes with high-amplitude.

One necessary condition for the presence of modal interaction is that the linear natural frequencies ω_i are commensurate or nearly commensurate, i.e., $\sum_{i=1}^{n} k_i\omega_i \approx 0$, with k_i being positive or negative integers [5, p. xvii]. However, for sandwich panels, this condition can be easily fulfilled and there may even exist numerous groups of linear modes which simultaneously fulfill this condition. As an example, Table 1 shows the first 15 linear natural frequencies of a flat sandwich panel experimentally investigated by [6]. Among these frequencies, the following combinations simultaneously fulfill the nearly commensurable condition: $\omega_{21}(45.0)+\omega_{41}(133.0)=178.0\approx\omega_{42}(177.0)$, $\omega_{12}(69.0)+\omega_{42}(177.0)=246.0=\omega_{14}(246.0)$, $\omega_{22}(92.0)+\omega_{23}(169.0)=261.0\approx\omega_{24}(262.0)$, $2\omega_{21}(90.0)+\omega_{31}(78.0)=168.0\approx\omega_{23}(169.0)$, $\omega_{12}(69.0)+\omega_{32}(129.0)=198.0\approx\omega_{33}(199.0)$, $\omega_{21}(45.0)+\omega_{13}(152.0)=197.0\approx\omega_{33}(199.0)$. This motivates us to make an in-depth study of the nonlinear vibration responses of sandwich panels.

To keep our effort to be focused, we consider in the present paper a straight two-dimensional (2-D) sandwich panel with transversely compressible core, which can be modeled as a sandwich beam, and investigate its nonlinear dynamic responses in the presence of combination internal resonance. Towards this end, an effective semi-analytical and semi-numerical method is used to investigate the modal interaction phenomena of the sandwich beam in the presence of combination internal resonance.

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Table 1
Eigenfrequencies ω_{mn} (Hz) of a flat sandwich panel.

m	n					Note
	1	2	3	4	5	
1	23.5	71.0	146.5	245.3	362.5	Present study
		69.0	152.0	246.0	381.0	[1] (exp.)
	23.0	71.0	146.0	244.0	360.0	[1] (num.)
2	45.1	92.1	166.7	264.5		Present study
	45.0	92.0	169.0	262.0		[1] (exp.)
	45.0	91.0	165.0	263.0		[1] (num.)
3	80.7	126.8	200.1			Present study
	78.0	129.0	199.0			[1] (exp.)
	80.0	126.0	195.0			[1] (num.)
4	130.0	174.9				Present study
	133.0	177.0				[1] (exp.)
	129.0	174.0				[1] (num.)
5	192.3					Present study
	188.0					[1] (exp.)
	191.0					[1] (num.)

Specifically, a nonlinear sandwich structures theory originally developed by Hohe and Librescu [7] is adopted to investigate the sandwich beam with transversely compressible core. The theory adopts the standard Kirchhoff hypothesis for the face sheets whereas a {2, 1}-order power series expansion is introduced for modeling the core's displacements. After incorporating the nonlinearity of deformation of the sandwich beam, the equations of motion and the compatible boundary conditions are derived from an extended Hamilton's principle [8]. Then, Galerkin's method and the method of multiple scales are adopted to semi-discretize and solve the related nonlinear problems. Next, modal interactions in the presence of combination internal resonance are analytically investigated and the solvability conditions are numerically integrated via the 4th-order Runge–Kutta method (see e.g., [8, pp. 214–222]). Finally, for the purpose of validating the preceding theoretical results, the commercial finite element code ABAQUS [9] is used to simulate the nonlinear vibration responses of the sandwich beam subjected to primary-resonance excitation. The fast Fourier transform (FFT) is further used for the frequency-domain analysis.

2. Kinematics and constitutive relations

We will consider a symmetric sandwich beam with respect to its mid-surface (see Fig. 1). The thickness of each of the face sheets and the core are denoted by t^f and t^c, respectively, while its length is denoted by ℓ. A Cartesian coordinate system x_2–x_3 is adopted as shown in Fig. 1.

For modeling the sandwich beam, we follow the theoretical framework provided in [7], in which, the displacements u_i (i=2,3) of the three layers of the sandwich beam are individually expanded into power series with respect to x_3. For the face sheets, the classical Kirchhoff hypothesis is adopted whereas the second-order power series expansion is used for the core layer's horizontal displacement, and the first-order power series expansion is used for the core layer's vertical displacement. It is termed as the {2,1}-order theory according to the name convention proposed in [10]. As a result, the following expressions for the displacement field of the face sheets and the core layer are postulated.

For the face sheets:

$$v_2^t = u_2^a + u_2^d - \left(x_3 + \frac{t^c + t^f}{2}\right)(u_{3,2}^a + u_{3,2}^d) \tag{1a}$$

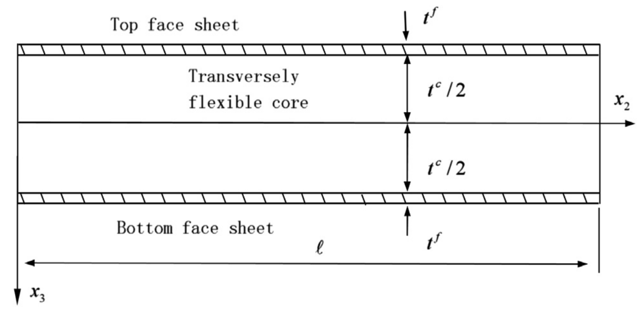

Fig. 1. Geometry of a sandwich beam.

$$v_2^b = u_2^a - u_2^d - \left(x_3 - \frac{t^c + t^f}{2}\right)(u_{3,2}^a - u_{3,2}^d) \tag{1b}$$

$$v_3^t = u_3^a + u_3^d, \quad v_3^b = u_3^a - u_3^d \tag{1c}$$

where u_2 and u_3 denote displacements in the x_2 and the x_3 directions of a point on the mid-surface, respectively, whereas v_2 and v_3 signify displacements of any point of the beam; t^c and t^f denote, respectively, the thickness of the core and of the face sheet. Superscripts a and d signify, respectively, the average and the half-difference of the top and the bottom face sheets' mid-surface displacements u_j^t and u_j^b. That is

$$u_j^a \equiv \frac{1}{2}(u_j^t + u_j^b), \quad u_j^d \equiv \frac{1}{2}(u_j^t - u_j^b), \ j = 2, 3 \tag{2a}$$

and subscripts t and b denote the top and the bottom face sheets, respectively. Furthermore, $u_{3,2} \equiv \partial u_3 / \partial x_2$.

For the core layer:

$$v_2^c = u_2^a - \frac{t^f}{2} u_{3,2}^d - \frac{2x_3}{t^c} u_2^d + \frac{t^f}{t^c} x_3 u_{3,2}^a + \left[\frac{4(x_3)^2}{(t^c)^2} - 1\right] \Omega_2^c \tag{3a}$$

$$v_3^c = u_3^a - \frac{2x_3}{t^c} u_3^d \tag{3b}$$

here, displacement function Ω_2^c describes the warping of the core. We note that Eqs. (2) and (3) involve five basic unknown functions, $u_2^a, u_2^d, u_3^a, u_3^d, \Omega_2^c$. The displacement field at points on the interfaces between face sheets and the core is assumed continuous.

To approximate the geometrical nonlinear effect, the beam's deformation is described in terms of the nonlinear Green–Lagrange strain tensor. The components of the strain tensor are given by

$$\varepsilon_{22} = v_{2,2} + \frac{1}{2}(v_{3,2})^2, \quad \varepsilon_{33} = v_{3,3} + \frac{1}{2}(v_{3,3})^2, \quad \gamma_{23} = v_{2,3} + v_{3,2} + v_{3,2} v_{3,3} \tag{4}$$

The sandwich beam theory presented above is not restricted to any kind of specific material model. Nevertheless, orthotropic materials are assumed for both face sheet and core layer. The core is further assumed to be weak, i.e., σ_{22}^c is negligible [3]. To the 2-D problem, we are addressing the stress–strain relations for the face sheets and the core can be represented as

$$\begin{Bmatrix} \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} Q_{22} & Q_{23} & 0 \\ Q_{23} & Q_{33} & 0 \\ 0 & 0 & Q_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \end{Bmatrix} \tag{5}$$

Introducing the concept of reduced stiffness (see e.g.,[11]), the stress–strain relations for the face sheets and the core can be summarized as

$$\sigma_{22}^f = \overline{Q}_{22}^f \varepsilon_{22}^f, \quad \sigma_{23}^f = Q_{44}^f \gamma_{23}^f \tag{6a}$$

$$\sigma_{33}^c = Q_{33}^c \varepsilon_{33}^c, \quad \sigma_{23}^c = Q_{44}^c \gamma_{23}^c \tag{6b}$$

where the reduced stiffness $\overline{Q}_{22}^f \equiv Q_{22}^f - [(Q_{23}^f)^2 / Q_{33}^f]$.