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Initial wrinkling and its evolution of membrane inflated cone in bending

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ABSTRACT

The concept of the wrinkling factor is firstly presented to obtain the wrinkling condition. An extremum method is then proposed to predict the critical wrinkling load and the initial wrinkling location by searching the maximum of the wrinkling factor. Here the critical wrinkling load is defined as the ratio of the wrinkling moment versus the initial wrinkling location, which is different from previous definition. The nondimensional analyses show that the critical wrinkling load and the initial wrinkling location are both closely related to the taper ratio of the inflated cone. The critical taper ratio is 1.5 which corresponds to the highest load-carrying efficiency of the inflated cone in bending. The wrinkled region is finally predicted to deeply understand the wrinkling evolution in the bended membrane inflated cone. A series of wrinkling experiments on the inflated cone in bending are performed to verify the accuracy and the validation of the proposed method. The good agreements between the tests and the predictions give confidence to use the extremum method for wrinkling analysis of the inflated load-carrying structures.

1. Introduction

The inflated booms have become increasing popular for a series of application in space inflatable membrane structures. Typical examples include inflatable wings, solar sails, truss structures and inflatable antennas [1–3] etc. The inflated booms are used to support and control the shape of inflatable antenna reflector. For solar sails, the inflated booms need be carefully designed to meet deployable requirements [1,2]. The multi-segmented inflated booms are mainly designed to carry loads and control the pneumatic shape of the large-sized flexible inflated wings [3]. With these applications, the inflated booms need to meet high axial-direction precision, load-carrying, and wrinkle-free requirements. Take inflatable antenna reflector as an example, an inflated Kapton boom with 5 m length, 2.5×10^{-2} m radius, 25×10^{-6} m wall thickness, and 10 KPa inflated pressure, needs to meet an 1 mm axial-direction precision [4]. However, the inflated booms made of membrane materials with basic properties in large but thin, lightweight and flexible, are very easy to be wrinkled. The inflated booms subject to bending were found to develop short wavelength periodic ripples on the compressed side, and the inflated booms buckled locally and collapsed soon after the appearance of the wrinkles [5,6]. Accurate evaluation of bending-wrinkling characteristics is important for better prediction of buckling load, vibration response and deflection of inflated booms.

For inflated beams, bending-wrinkling behavior may be divided into problems in which the wall material is regarded as either a true membrane or a thin-shell, which result in two models. The first of these two models may be called "membrane model" [7–11]. The second is named as "thin-shell model" [4,12–18]. The distinctions between these two models are depending on whether the bending and compression stiffness of the wall material are considered or not. For "membrane model", the membrane with zero bending stiffness cannot resist any compression loads or bending moments. For "thin-shell model", the wall material is considered as a thin-shell with a small but non-zero bending stiffness. In fact, the "membrane model" corresponds to the case of zero critical compressive stress in the "thin-shell model" [12,13,15].

The research on inflated beams is mainly focused on load deflection behavior in pre- and post-wrinkling situations. It has established a thorough understanding of pre- and post-wrinkled behavior of inflated beams in bending. However, little work has been done in study of the bending and wrinkling behaviors of the inflated cones. The inflated cone is regarded as the optimum geometry of the straight cylindrical boom, and has the larger possible structural efficiency and load-carrying ability [19]. Several researches on the inflated cones are mainly focused on the predictions on the wrinkling and collapsed moments. These predictions were also compared with the bending experimental results of inflated cones [17,18].

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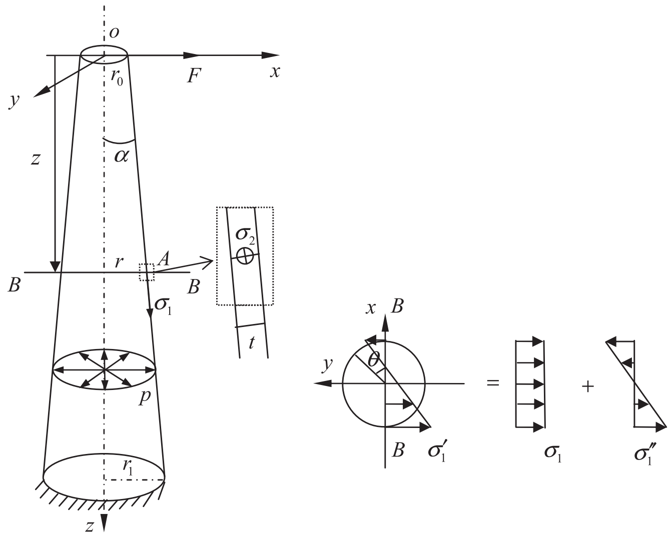

Fig. 1. Inflated conical cantilever beam in bending.

Increasing use of inflated cones in aerospace application has spurred a need in deeply and accurately evaluating structural properties of inflated conical beams in bending. The purpose of this paper is to provide a method for accurately predicting wrinkling characteristics of inflated cone in bending. We will lay a special stress on predictions of the wrinkling condition, the critical wrinkling load, the initial wrinkling location and the wrinkling evolution of membrane inflated cone in bending. The wrinkling tests on the membrane inflated cone are used to verify the predictions.

2. Mechanics of inflated cone in bending

An inflated core subjected to tip bending is shown in Fig. 1. r_0 and r_1 are the free-end and the fixed-end radius of the inflated cone, respectively. α is the half cone angle. r is the cross-sectional radius at point A. t is the wall thickness of the inflated cone. p is the inflated pressure.

Here, r_0 is assumed to be constant. Thus, the radius of the inflated cone r can be expressed as a function of the free-end radius r_0 and the axial coordinate z.

$$r = z\tan\alpha + r_0 \quad (0 \le z \le z_0) \tag{1}$$

Here, z_0 is the total height of the inflated cone.

$$z_0 = (r_1 - r_0)\ \cot\alpha \tag{2}$$

When the tip transverse load F is not taken into account, the force equilibrium of the inflated cone is given by

$$\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2} = \frac{p}{t} \tag{3}$$

where, the subscripts 1 and 2 denote the axial and hoop coordinates, respectively. At point A, the axial stress σ_1 is given by

$$\sigma_1 = \frac{pr}{2t\cos\alpha} \tag{4}$$

At point A, the axial coordinate is z, the moment M of the inflated cone under tip transverse load F is obtained as

$$M = Fz \tag{5}$$

The wrinkles will be formed when the moment at point A reaches a critical value. The moment corresponding to the first wrinkle is defined as the wrinkling moment. Based on the wrinkling criterion for isotropic membrane [7,10], the wrinkling condition is given by

$$\sigma_{min} = 0 \tag{6}$$

As shown in Fig. 1, the axial stress resultant of the inflated cone σ_1' can be decomposed into two parts: the stress σ_1 due to inflated pressure p and the stress σ_1'' due to the tip transverse load F. Thus the axial stress resultant of the inflated cone can be expressed as

$$\sigma_1' = \sigma_1 + \sigma_1'' \tag{7}$$

Here, the stress σ_1'' due to the tip transverse load F can be written as

$$\sigma_1'' = C_0\cos\theta \tag{8}$$

where, C_0 is an undetermined constant.

Then we have

$$\sigma_1' = \frac{pr}{2t\cos\alpha} + C_0\cos\theta \tag{9}$$

C_0 can be determined according to the wrinkling stress criterion (Eq. (6)). The axial stress resultant σ_1' is then obtained as

$$\sigma_1' = \frac{pr}{2t\cos\alpha}(1 - \cos\theta) \tag{10}$$

The moment equilibrium at point A (the axial coordinate z) is then written as

$$M - r^2 t\int_0^{2\pi} \sigma_1' \cos\alpha\cos\theta d\theta = 0 \tag{11}$$

Thus, the wrinkling moment M_w can be obtained by substituting Eq. (10) into Eq. (11)

$$M_w = M|_{\theta=0} = \frac{\pi p r^3}{2} \tag{12}$$

3. Predictions on the wrinkling characteristics

3.1. Wrinkling condition

The wrinkles will be formed when the moment at point A reaches the wrinkling moment. The concept of the wrinkling factor is defined as the ratio of the moment M at point A versus the wrinkling moment M_w. According to the definition of the wrinkling factor, the wrinkles will occur when $\lambda \ge 1$, which responds to the wrinkling condition.

$$\begin{cases} \lambda \ge 1 & \textit{wrinkling} \\ \lambda < 1 & \textit{no wrinkling} \end{cases} \tag{13}$$

The wrinkling factor is a function of the axial coordinate z because the moment is a function of the axial coordinate. The wrinkling factor can be expressed as

$$\lambda = \frac{M}{M_w} = \frac{2Fz}{p\pi r^3} \tag{14}$$

where, r is expressed in Eq. (1).

3.2. Initial wrinkling location

Based on the wrinkling condition (Eq. (13)), the wrinkles will be firstly formed when the wrinkling factor reaches its maximum. Thus, the initial wrinkling location can be indirectly obtained by searching the extremum (the maximum) of the wrinkling factor, which is named as the extremum method.