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Analysis of resonance effect for a railway track on a layered ground

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ABSTRACT

When a train runs on soft ground it can approach or even exceed the speed of surface waves in the ground. Under such conditions the amplitudes of the track response increase considerably. Moreover, a resonance-like phenomenon can occur in which a clear oscillation trail can be observed behind the moving axle loads. An investigation is presented of this resonance frequency and the critical speed effect for a track on a layered half-space subject to a moving load. Three different methods are used to investigate this resonance frequency: (i) the spectrum of the response to a moving load, (ii) analysis of the dispersion curves of the ground, and (iii) frequency analysis of the response to a stationary load. A parameter study is presented of a layered half-space ground with different P-wave speeds, S-wave speeds, and depth of the upper layer. The critical speeds are found in each case; in such a layered ground, the critical speed is greater than the Rayleigh wave speed of the soft upper layer due to the influence of the underlying half-space. The oscillating frequencies are shown to vary with the speed of the moving load, tending to reduce when the load speed increases. The P-wave speeds of both the upper layer and the underlying half-space are found to have negligible influence on the critical velocity and on the oscillating frequency; the S-wave speed of the half-space has only a small influence. Larger differences are found when the depth of the layer is varied. Finally, a formula for calculating this resonance frequency is proposed.

Introduction

High-speed trains have become an important means of public transport due to their efficiency and relatively low impact on the environment. However, with increasing train speed, especially when the track alignment crosses soft ground, the track deflections increase dramatically as the train speed approaches the speed of surface waves in the soil [1,2]. Furthermore, when the speed of a load moving on a layered ground exceeds the Rayleigh wave speed of the upper layer, a resonance phenomenon is observed in which the track and ground surface behind the load oscillates with a certain frequency [1,3–5]. These phenomena were also found in the site measurements at Ledsgård in Sweden [6]. To avoid these effects, costly soil improvement or additional foundation structures may be required such as strengthening the embankment [7] or ground stabilisation using lime cement columns [8].

For an ideal homogeneous soil the critical speed is readily determined as the Rayleigh wave speed of the soil [9]. For a layered ground the phenomenon is more complex, but nevertheless a number of authors have investigated the critical speed effect and methods to evaluate it [9–14]. However, only limited attention has been given to the resonance-like phenomenon that occurs in some situations for a layered half-space.

Even in cases where the moving load speed is lower than the critical speed, significant vibration may occur that is associated with this resonance-like frequency, especially if it coincides with a strong excitation frequency from the train, for example due to the axle-passing frequencies [15]. Ground-borne vibration has become an important issue due to environmental concerns and, to give a correct assessment of the vibration levels occurring while train is passing, a numerical model that can correctly characterise the dynamic behaviour is required. Soil damping is an important factor for modelling the ground-borne vibration and in time domain models Rayleigh damping is commonly used for soil properties [12,16,17]. To apply the Rayleigh damping model, as used by Shih et al. [16], an appropriate 'dominant' frequency is required to select the corresponding damping coefficients. This is important not only to ensure the wave energy is sufficiently attenuated at the boundary of the model but also to represent the soil damping behaviour more correctly. Better understanding of this resonance phenomenon can thus lead to a more representative soil damping model and, together with the appropriate excitation mechanisms, to a better assessment for the ground-borne vibration.

Wave propagation in a homogeneous half-space is non-dispersive and consequently no resonance frequency is found for a moving point source [16]. In contrast, a resonance frequency can be found for layered half-space soil. This resonance frequency is usually identified with the

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'cut-on frequency' of the upper layer, above which the waves begin to propagate in the upper soil layer [18]. It is indicated in the literature that the resonance frequency is related to the depth of the layer and the P-wave speed in the first layer [17–19]. A modified formula, based on the shear wave speed of the first layer and its depth with a cut-off factor, was introduced by Mehzer et al. [12]. However, the effect of load motion, which may modify the frequency, is not discussed in these papers.

In this paper an investigation is carried out into this resonance phenomenon and its dependence on the properties of a layered ground. This analysis is carried out by using a three-dimensional semi-analytical method [1,5]. This model is based on the formulation of Haskell and Thomson [20,21] and uses a two-dimensional Fourier transform over the axial and transverse coordinates to represent the layered ground in terms of the corresponding wavenumbers. The track is represented by a layered beam structure coupled to the ground over a finite-width strip. A static or moving load can be considered which may be either constant or harmonically varying.

Herein three different methods are used to study this resonance frequency for a track on a layered half-space. First, the semi-analytical method presented in [5] is used to calculate the rail receptance due to a stationary harmonic load applied on the rail. Second, a quasi-static load moving along the track is considered using the same model as in [1]. The frequency spectra of the response in the non-moving frame due to the moving load are obtained and used to identify the resonance frequency. Third, following the method in [10], dispersion curves from the layered half-space with and without the track are calculated and used to estimate the resonance frequency for different load speeds by finding the intersection points between the dispersion curves and the load speed line.

Results are presented for a layered half-space in which various values are considered for the depth of the upper ground layer and the P- and S-wave speeds of the layer and the substratum; results are also compared with those for a homogeneous half-space. In all cases the upper layer is considered to be softer than the underlying half-space, as is commonly found in practice. The critical speed is determined first for each case from the semi-analytical model. Then the resonance frequency is assessed using the three different methods and the results are compared. Finally, the results from the above methods are compared with the results from the formulae indicated in [12,17,18] and a revised formula is proposed.

Parameters used in the study

A range of different cases are introduced here for a ground with a single soft layer above a stiffer half-space. The parameters defining these cases are listed in Table 1. These are the P-, and S-wave speeds for the upper layer and the underlying half-space, as well as the depth of the first layer. The S-wave speed of the upper layer is kept fixed at 60 m/s and the other wave speeds are varied relative to this. The soil density is set to 2000 kg/m^3 throughout. The damping is represented by a constant damping loss factor of 0.05 in each case.

Table 1
Parameters used to define the ground.

No.	layer depth, H (m)	1st layer P-wave, c_{p1} (m/s)	1st layer S-wave, c_{s1} (m/s)	2nd layer P-wave, c_{p2} (m/s)	2nd layer S-wave, c_{s2} (m/s)	Critical speed, V_{cr} (m/s)
1	2	120	60	240	120	79
2	2	240	60	240	120	80
3	2	240	60	480	120	80
4	2	240	60	480	240	82
5	2	120	60	170	85	70
6	4	120	60	240	120	62
7	8	120	60	240	120	57
8	∞	120	60	–	–	55

Table 2
Track properties (for two rails).

Parameter	Value	Units
Rail mass	120	kg/m
Rail bending stiffness	1.28×10^7	Nm^2
Rail damping loss factor	0.01	
Railpad stiffness per unit track length	1.0×10^9	N/m^2
Railpad damping loss factor	0.1	
Sleeper mass per unit track length	541.8	kg/m
Ballast stiffness per unit track length	4.64×10^9	N/m^2
Ballast mass per unit track length	1740	kg/m
Ballast damping loss factor	0.04	
Ballast width at the bottom	3.2	m

The chosen reference soil properties for the upper layer corresponds to very soft soil, which approximately represents a typical soft clay. Although in such a situation the soil may have a high water content, the main focus of the present work is to look generically at how the wave speeds of the soil influence the dynamic behaviour of the layered ground. The wave speeds have therefore been chosen somewhat arbitrarily for convenience in the parametric analysis. The actual values of, for example, the mass density are less important for the present study.

In the initial case, the P-wave speed of the upper layer is set to 120 m/s and the P- and S-wave speeds of the substratum are assumed to be double the values in the upper layer. This case is used as the reference case for comparison with the others. In cases 2 and 3, the P-wave speeds of the two layers are varied whereas in cases 4 and 5, the S-wave speed of the underlying half-space is varied while keeping the P-wave speed equal to twice the value of the S-wave speed. In cases 6 and 7, the layer depth is varied while keeping the same wave speeds as case 1. The homogeneous half-space (case 8) is an extreme case with an infinitely deep upper layer.

The track properties used in each case are listed in Table 2 and are largely based on the ballasted track used in [22]. The value for the rail pad stiffness is equivalent to a stiffness of 300 MN/m per pad, which corresponds to a medium stiffness rail pad. In the current model, the railway track is considered as a straight ballasted track at the surface of the layered elastic half-space, as shown in Fig. 1(a). Linear dynamic behaviour is assumed throughout. The track is considered invariant in the longitudinal (x) direction and is modelled as a beam supported by vertical springs representing the rail pads, a layer of mass representing the sleepers and a further layer of springs with consistent mass representing the ballast, as shown in Fig. 1(b). Since each axle load is distributed equally between the two rails, a single beam is used to represent both rails.

The model is formulated in the wavenumber-frequency domain and uses the transfer function matrices for the ground formulated in [5] and [1] in a frame of reference moving with the loads. For the calculation of the response in the wavenumber domain, equally-spaced wavenumbers are used for the directions both parallel and normal to the track. In each direction the maximum wavenumber is set to 10π rad/m and the number of wavenumber points in each direction is set to be 2048. The inverse Fourier transform is carried out using the FFT algorithm in order to transform the response from the wavenumber to the space domain. The maximum wavenumber and the wavenumber discretization define spatial resolution and the sizer of the spatial domain and are chosen to ensure an efficient and sufficiently accurate Fourier transform.

Investigation of the critical speed

To determine the critical speed, the maximum displacement on the track induced by a moving point load is calculated by using the three-dimensional semi-analytical track/ground model of [1]. The results for