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# Stochastic frontier estimation of budgets for Kuhn–Tucker demand systems: Application to activity time-use analysis

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ABSTRACT

We propose a stochastic frontier approach to estimate budgets for the multiple discrete–continuous extreme value (MDCEV) model. The approach is useful when the underlying time and/or money budgets driving a choice situation are unobserved, but the expenditures on the choice alternatives of interest are observed. Several MDCEV applications hitherto used the observed total expenditure on the choice alternatives as the budget to model expenditure allocation among choice alternatives. This does not allow for increases or decreases in the total expenditure due to changes in choice alternative-specific attributes, but only allows a reallocation of the observed total expenditure among different alternatives. The stochastic frontier approach helps address this issue by invoking the notion that consumers operate under latent budgets that can be conceived (and modeled) as the maximum possible expenditure they are willing to incur. The proposed method is applied to analyze the daily out-of-home activity participation and time-use patterns in a survey sample of non-working adults in Florida. First, a stochastic frontier regression is performed on the observed out-of-home activity time *expenditure* (OH-ATE) to estimate the unobserved out-of-home activity time *frontier* (OH-ATF). The estimated *frontier* is interpreted as a subjective limit or maximum possible time individuals can allocate to out-of-home activities and used as the time budget governing out-of-home time-use choices in an MDCEV model. The efficacy of this approach is compared with other approaches for estimating time budgets for the MDCEV model, including: (a) a log-linear regression on the total observed expenditure for out-of-home activities and (b) arbitrarily assumed, constant time budgets for all individuals in the sample. A comparison of predictive accuracy in time-use patterns suggests that the stochastic frontier and log-linear regression approaches perform better than arbitrary assumptions on time budgets. Between the stochastic frontier and log-linear regression approaches, the former results in slightly better predictions of activity participation rates while the latter results in

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slightly better predictions of activity durations. A comparison of policy simulations demonstrates that the stochastic frontier approach allows for the total out-of-home activity time expenditure to either expand or shrink due to changes in alternative-specific attributes. The log-linear regression approach allows for changes in total time expenditure due to changes in decision-maker attributes, but not due to changes in alternative-specific attributes.



## 1. Introduction

Numerous consumer choices are characterized by "multiple discreteness" where consumers can potentially choose multiple alternatives from a set of discrete alternatives available to them. Along with such discrete-choice decisions of which alternative(s) to choose, consumers typically make continuous-quantity decisions on how much of each chosen alternative to consume. Such multiple discrete–continuous (MDC) choices are being increasingly recognized and analyzed in a variety of social sciences, including transportation, economics, and marketing.

A variety of approaches have been used to model MDC choices. Among these, an increasingly popular approach is based on the classical microeconomic consumer theory of utility maximization. Specifically, consumers are assumed to optimize a direct utility function $U(\mathbf{t})$ over a set of non-negative consumption quantities $\mathbf{t} = (t_1, \ldots, t_k, \ldots, t_K)$ subject to a budget constraint, as below:

$$Max\ U(\mathbf{t}) \text{ such that } \sum_{k=1}^{K} p_k t_k = y \text{ and } t_k \geqslant 0 \quad \forall k = 1, 2, \ldots, K \tag{1}$$

In the above equation, $U(\mathbf{t})$ is a quasi-concave, increasing, and continuously differentiable utility function of the consumption quantities, $p_k$ ($k = 1, 2, \ldots, K$) are unit prices for all goods, and $y$ is a budget for total expenditure. A particularly attractive approach for deriving the demand functions from the utility maximization problem in Eq. (1), due to Hanemann (1978) and Wales and Woodland (1983), is based on the application of Karush–Kuhn–Tucker (KT) conditions of optimality with respect to the consumption quantities. When the utility function is assumed to be randomly distributed over the population, the KT conditions become randomly distributed and form the basis for deriving the probability expressions for consumption patterns. Due to the central role played by the KT conditions, this approach is called the KT demand systems approach (or KT approach, in short).

Over the past decade, the KT approach has received significant attention for the analysis of MDC choices in a variety of fields, including environmental economics (von Haefen and Phaneuf, 2005), marketing (Kim et al., 2002), and transportation. In the transportation field, the multiple discrete–continuous extreme value (MDCEV) model formulated by Bhat (2005, 2008) has lead to an increased use of the KT approach for analyzing a variety of choices, including individuals' activity participation and time-use (Habib and Miller, 2008; Pinjari et al., 2009; Chikaraishi et al., 2010; Eluru et al., 2010; Spissu et al., 2009; Sikder and Pinjari, 2014), household vehicle ownership and usage (Ahn et al., 2008; Jaggi et al., 2011; Sobhani et al., 2013; Faghih-Imani et al., 2014), recreational/leisure travel choices (von Haefen and Phaneuf, 2005; Van Nostrand et al., 2013), energy consumption choices, and builders' land-development choices (Farooq et al., 2013; Kaza et al., 2012). Thanks to these advances, KT-based MDC models are being increasingly used in empirical research and have begun to be employed in operational travel forecasting models (Bhat et al., 2013a). On the methodological front, recent literature in this area has started to enhance the basic formulation in Eq. (1) along three specific directions: (a) toward more flexible, non-additively separable utility functions that accommodate rich substitution and complementarity patterns in consumption (Bhat et al., 2013b), (b) toward more flexible stochastic specifications for the random utility functions (Pinjari and Bhat, 2010; Pinjari, 2011; Bhat et al., 2013c), and (c) toward greater flexibility in the specification of the constraints faced by the consumer (Castro et al., 2012).

### *1.1. Gaps in research*

Despite the methodological advances and many empirical applications, one particular issue related to the budget constraint has yet to be resolved. Specifically, almost all KT model formulations in the literature, including the MDCEV model, assume that the available budget for total expenditure, i.e., $y$ in Eq. (1), is fixed for each individual (or for each choice occasion, if repeated choice data is available). Given the fixed budget, any changes in the decision-maker characteristics, choice alternative attributes, or the choice environment can only lead to a reallocation of the budget among different choice alternatives. The formulation itself does not allow either an increase or a decrease in the total available budget. Consider, for example, the context of households' vehicle holdings and utilization. In most applications of the KT approach for this context (Bhat et al., 2009; Ahn et al., 2008), a total annual mileage budget is assumed to be available for each household. This mileage budget is obtained exogenously for use in the KT model, which simply allocates the given total mileage among different vehicle types. Therefore, any changes in household characteristics, vehicle attributes (e.g., prices and fuel economy) and gasoline prices can only lead to a reallocation of the given mileage budget among the different vehicle types without