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Semi-analytical solution for groundwater ingress into lined tunnel

Hong-wei Ying, Cheng-wei Zhu*, Hua-wei Shen, Xiao-nan Gong

Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China
Key Laboratory of Soft Soils and Geoenvironmental Engineering of Ministry of Education, Zhejiang University, Hangzhou 310058, China

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ABSTRACT

A semi-analytical solution for the groundwater ingress into a lined tunnel in a semi-infinite aquifer is derived based on the conformal mapping technique. The new solution considers the property of the lining, such as the permeability coefficient of the lining as well as the external and internal lining radii. A numerical model is established using the software COMSOL to verify the semi-analytical solution, and a good agreement is found. Parameters including water layer thickness, burial depth of the tunnel, permeability coefficient of the lining and the lining thickness are discussed in detail. This paper explains why the water ingress obtained from early research should be reduced by Heuer's empirical factor 1/8. Moreover, an optimal burial depth is found when the permeability of the lining is close to the permeability of the aquifer.

1. Introduction

Water ingress is a key issue affecting the construction and operating phases of drained tunnels (Anagnostou, 1995; Arjnoi et al., 2009; Kong, 2011). Moreover, some researchers have determined that most tunnels eventually act as drains (Atkinson and Mair, 1983; Wongsaroj et al., 2007). Therefore, many studies are devoted to estimating the accurate water ingress into tunnels mainly using analytical and numerical methods (Nam et al., 2007; Park et al., 2008a; Shin et al., 2002, 2011).

Early researchers (Goodman et al., 1965; Polubarinova-Kochina, 1962) derived approximate expressions for the water ingress into deeply buried tunnels. Lei (1999) acquired a solution without the assumption of large burial depth based on the image method (Harr, 1962). Joo and Shin (2014) studied the relationship between water pressure and water ingress into underwater tunnels for laminar and turbulent flows. In recent literature, another method alleged that conformal mapping could be used to investigate different boundary conditions along the tunnel circumference (El Tani, 2003; Kolymbas and Wagner, 2007). Huangfu et al. (2010) validated these analytical solutions with the software FLAC3D. A revisit was given by Park et al. (2008b) regarding the analytical solutions based on the conformal mapping in this field.

However, nearly all the aforementioned literature regards the lining layer as the isopotential surface, which is too simplified when considering lining properties such as the lining thickness and permeability. This may be the reason why the water ingress prediction from earlier research is usually larger than practical measurement. For instance, Raymer (2001) found the Goodman equation overestimated tunnel inflows after reviewing a number of tunneling case studies and proposed an equation for estimating tunnel water inflow comprising a modified version of the Goodman equation, where a reduction factor, i.e. 1/8, is applied. With the purpose of deriving an accurate solution for water ingress, this paper considers lined tunnels within a semi-infinite aquifer as a double stratum model based on the conformal mapping technique. A numerical simulation is conducted to verify the solution. A parameter analysis, including the lining permeability, burial depth and tunnel radius, is discussed.

2. Mathematical statement

2.1. Basic assumptions

As shown in Fig. 1, a circular lining (domain II) with external and internal radii denoted as R and r, respectively, is buried in a semi-infinite aquifer (domain I). Here, burial depth h is defined as the distance between the center of the tunnel and the mud line. The aquifer is covered by the water layer with a height of H, and the mud line is chosen as the elevation reference datum. Additionally, this paper is based on the following assumptions.

(1) The aquifer is homogeneous with isotropic permeability;
(2) The flow is in a steady state and is governed by Darcy's law;
(3) The pore pressure on the inner circumference of the lining u is constant.

Assumption (1) may seem to be a strong simplification because of

* Corresponding author at: Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China.
E-mail addresses: ice898@zju.edu.cn (H.-w. Ying), zhuchengwei@zju.edu.cn (C.-w. Zhu), haywireshen@zju.edu.cn (H.-w. Shen), 13906508026@126.com (X.-n. Gong).

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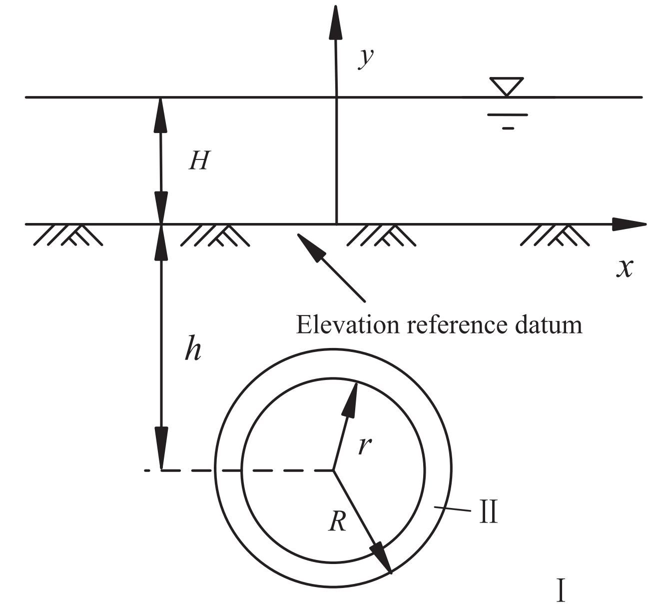

Fig. 1. Schematic diagram of underwater tunnel.

inhomogeneous distribution of hydraulic conductivity in the aquifer, but such process is still reasonable in terms of water ingress prediction as a pilot study (Kolymbas and Wagner, 2007).

2.2. Governing equation

According to the Darcy's law and mass conservation as well as the aforementioned assumptions, the governing equation for the seepage field in this problem is the Laplace equation, as shown below

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \tag{1}$$

where ϕ is the hydraulic head, equal to the sum of pressure and elevation heads, i.e.

$$\phi=\frac{p}{\gamma_w}+y \tag{2}$$

where p is the water pressure and γ_w is the unit weight of water.

2.3. Boundary conditions

From Section 2.1, two boundary conditions can be obtained, which are BC1:

$$\phi|_{y=0}=H \tag{3}$$

and BC2:

$$\phi|_{x^2+(y+h)^2=r^2}=h_u+y \tag{4}$$

where h_u, equal to u/γ_w, is the pressure head inside the lining layer.

3. Semi-analytical solution

The method of conformal mapping can facilitate the derivation of the hydraulic head in this study (Cao, 2014; Verruijt, 1997). As shown in Fig. 2, the semi-infinite domain I in plane – Z is mapped as a ring domain in plane – ζ with the internal and external radii, α and 1, respectively, based on the complex mapping function in Eq. (5).

$$\zeta=\xi+\eta i=\frac{z+ia}{z-ia} \tag{5}$$

where $a=\sqrt{h^2-R^2}$, and ξ and η are the Cartesian coordinates of an arbitrary point in plane – ζ.

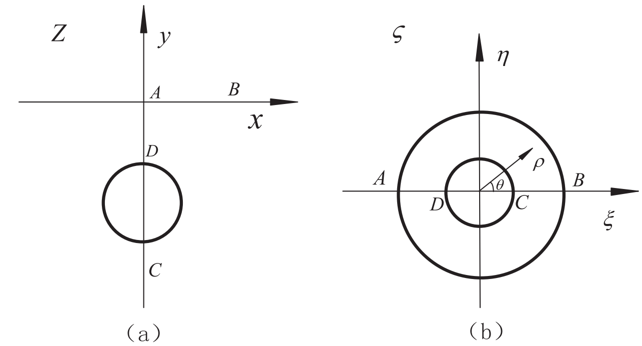

Fig. 2. Conformal mapping.

According to the property of this technique, the governing equation in the aquifer can be rewritten as shown below

$$\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial\eta^2}=0 \tag{6}$$

The general solution for Eq. (6) in domain I in plane – ζ in terms of polar coordinate is shown as follows (Park et al., 2008b):

$$\phi_{\rm I}=H+C_1\ln\rho_{\rm I}+\sum_{n=1}^{\infty}C_{2n}(\rho_{\rm I}{}^{n}-\rho_{\rm I}{}^{-n})\cos n\theta_{\rm I} \tag{7}$$

where $\rho_{\rm I}=\frac{\sqrt{(x^2+y^2-a^2)^2+4a^2x^2}}{x^2+(y-a)^2}$, and $\theta_{\rm I}=\arccos\frac{x^2+y^2-a^2}{\sqrt{(x^2+y^2-a^2)^2+4a^2x^2}}$.

Similarly, the general solution for Eq. (1) in domain II can be given as

$$\phi_{\rm II}=A_0+A_1\ln\rho+\sum_{n=1}^{\infty}(A_{2n}\rho^n+B_{2n}\rho^{-n})\cos n\theta \tag{8}$$

Eq. (8) has the following two boundary conditions at $\rho=r$ or R in the form of Fourier series based on Eqs. (4) and (7),

$$\phi_{II}(r,\theta)=\sum_{n=0}^{\infty}W_n\cos n\theta \tag{9}$$

$$\phi_{II}(R,\theta)=\sum_{n=0}^{\infty}V_n\cos n\theta \tag{10}$$

where $W_n=\begin{cases}-h+h_u & n=0;\\ r & n=1;\\ 0 & n\geqslant 2\end{cases}$; $V_n=\begin{cases}\frac{1}{2\pi}\int_0^{2\pi}\phi_I d\theta & n=0\\ \frac{1}{\pi}\int_0^{2\pi}\phi_I\cos n\theta d\theta & n\geqslant 1\end{cases}$. According to the theory of Fourier series, the coefficients before Eq. (8) and Eqs. (9) and (10) should be the same, so A_{2n} and B_{2n} have the following expressions

$$A_0=W_0-\frac{(V_0-W_0)\ln r}{\ln R/r};\ A_1=\frac{V_0-W_0}{\ln R/r};\ A_{2n}=\frac{V_nR^n-W_nr^n}{R^{2n}-r^{2n}};\ B_{2n}=\frac{V_nR^{-n}-W_nr^{-n}}{R^{-2n}-r^{-2n}} \tag{11}$$

Note that there are $n+1$ unknowns, i.e., C_1 and C_{2n}, in Eqs. (7)–(11), so $n+1$ equations are needed to calculate the values of these unknowns. The seepage continuity condition at the interface between the tunnel and aquifer can offer such an equation set, as shown in Eq. (12).

$$\frac{\partial\phi_I}{\partial\rho}=\frac{k_l}{k_s}\frac{\partial\phi_{II}}{\partial\rho} \tag{12}$$

where k_l and k_s are the hydraulic conductivities of the lining and aquifer, respectively.

The left side of Eq. (12) can be expanded as the Fourier series with the coefficient for each term as shown below.