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Modeling the progressive failure of hard rock pillars

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ABSTRACT

An enhanced cohesion weakening and friction strengthening model is proposed to simulate the progressive failure of hard rock pillars using two- and three-dimensional finite difference analysis. The documented behavior of 85 pillars in two mines was used for model validation. The numerical models successfully separated unstable/failed pillars from stable pillars and also captured the observed mechanism of progressive failure in hard rock pillars. Pillar stress-strain curves indicated that while pillars with $W/H < 2$ exhibit strain softening, wider pillars begin to display strain hardening. It was shown that strength of wide pillars may be overestimated by conventional models using empirically estimated parameters.

1. Introduction

Estimating the strength of pillars is a necessary step in the design of underground mines. In conventional mining methods such as room and pillar operations, economic incentives point towards higher excavation ratios by leaving smaller pillars between the stopes while safety and stability requirements favor wider stronger pillars. In some mining methods such as caving operations, optimal design of pillars is critical as not only pillars too narrow can jeopardize mine safety and stability but also pillars too wide can block the flow of ore through drawpoints. Therefore, pillar design poses a somewhat unique challenge to rock engineers where simple conservative approaches frequently used in other applications are no longer acceptable.

The most straight-forward method for estimation of pillar strength is using empirical formulas based on the analysis of large number of pillars in different mines. As in any empirical method, however, it is essential to note the conditions and ranges of variables in the data sets used for developing the formulas. Many of the pillars in the empirical data sets have limited width to height ratios and occur at relatively shallow depths in high quality rock masses (Lunder and Pakalnis, 1997; Martin and Maybee, 2000; Kaiser et al., 2011). Extending the application of empirical formulas beyond these ranges is unwarranted. In addition, the approach is very simplified and the effect of many parameters such as the condition of roof and floor, pillar-foundation interface and in situ stress ratio cannot be explored by the empirical formulas. Finally, these formulas only estimate the pillar strength while knowledge of the pre- and post-peak behavior is also important in mining operations.

Another approach to pillar design is using numerical modeling. This method can take into account most of the variables and complexities affecting pillar behavior. In addition, the complete pillar behavior, from initial loading to post-peak, can be captured. However, realistic modeling of pillars requires a comprehensive knowledge of material behavior and implementing a representative material model. Behavior of rocks can be described using mechanistic models (analytically derived from the principles of fracture and damage mechanics) or phenomenological models (empirically derived to describe experimental observations). Examples of mechanistic models are given in Griffith (1920, 1924), McClintock and Walsh (1962), Zhou et al. (2004, 2008), Zhou (2004), Zhou and Yang (2007), Abou-Chakra Guery et al. (2008), Bui et al. (2017). Phenomenological models such as those presented in this study are commonly used in practical design and analysis of large scale rock engineering structures.

Iannacchione (1989) and Whyatt and Board (1991) made early attempts to use strain softening models in finite difference analysis to explore the behavior of pillars. Martin and Maybee (2000) adopted brittle parameters in a finite element analysis to predict the strength of rock pillars. Duncan Fama et al. (1995) and Adhikary et al. (2002) used strain softening models in finite difference and finite element analyses to explore the behavior of coal pillars. Mortazavi et al. (2009) used the finite difference method and strain softening model to capture the behavior of hard rock pillars. Elmo and Stead (2010) used a hybrid finite element/discrete element method to explicitly model the fracture network inside pillars. Kaiser et al. (2011) presented a critical review of the common models and used an s-shaped failure envelope in finite element analysis of rock pillars.

The focus of this paper is to model the behavior of hard rock pillars using a Cohesion Weakening and Friction Strengthening (CWFS) model.

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Table 1
Empirical formulas for hard rock pillar strength, σ_p (W and H are the width and height of pillar in metres, respectively, σ_c is the uniaxial compressive strength of intact rock and κ is the mine pillar friction term.

Reference	Pillar strength, σ_p	σ_c (MPa)	No. of pillars	Rock mass
Hedley and Grant (1972)	$133\,(W^{0.5}/H^{0.75})$	230	28	Quartzite
von Kimmelmann et al. (1984)	$65\,(W^{0.46}/H^{0.66})$	94	57	Metasediments
Krauland and Soder (1987)	$35.4\,[0.778 + 0.222(W/H)]$	100	14	Limestone
Potvin et al. (1989)	$0.42\sigma_c(W/H)$	–	23	Canadian shield
Sjoberg (1992)	$74\,[0.778 + 0.222(W/H)]$	240	9	Limestone/Skarn
Lunder and Pakalnis (1997)	$0.44\sigma_c(0.68 + 0.52\kappa)$	–	178	Hard rocks

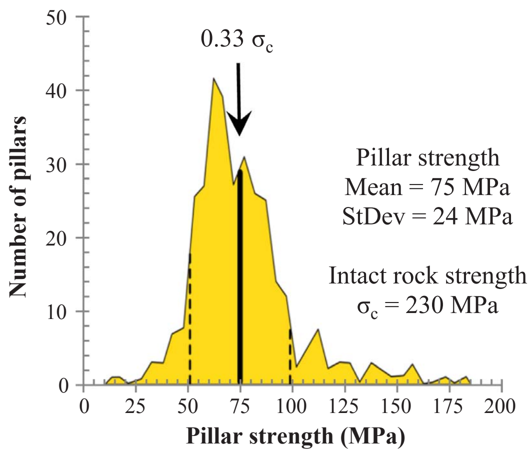

Fig. 1. Distribution of pillar strength at the Quirke mine, after (Swan et al., 1985).

Empirical formulas and factors affecting pillar strength are reviewed. The merits of a CWFS model in simulating progressive failure of hard rocks are illustrated. An enhanced CWFS is implemented in the finite difference code FLAC3D (Itasca Inc., 2009) to model hard rock pillars. Documented behavior of pillars at the Elliot Lake and Selebi-Phikwe mines is used to evaluate the proposed modeling approach.

2. Strength of hard rock pillars

Following the failure of Coalbrook coal mine in 1960, extensive research was initiated in South Africa to establish the strength of coal pillars. One of the early works is due to Salamon and Munro (1967) who analyzed 125 cases of coal pillar failure and expressed the strength of pillar as a power function of its width and height. Their findings were used by Hedley and Grant (1972) who analyzed 28 hard rock rib pillars at the Elliot Lake uranium mines and proposed a similar power function with modified exponents. Alternative pillar strength formulas were proposed by von Kimmelmann et al. (1984), Krauland and Soder (1987), Potvin et al. (1989) and Sjoberg (1992). Lunder and Pakalnis (1997) later compiled perhaps the most extensive data base of 178 hard rock pillars and suggested a new pillar strength formula. Table 1 summarizes empirical formulas that have been reported for the design of hard rock pillars. As evident from Table 1 and pointed out by Bieniawski (1992) and Lunder and Pakalnis (1997), pillar strength is influenced by size effect and shape effect.

2.1. Size effect

Size effect refers to the reduction of strength by increasing the size of the test sample. It is a fundamental characteristic of heterogeneous materials and is caused by the increasing number of weaker and softer elements within larger samples. As suggested by Bieniawski (1968) and Martin et al. (2012), there is a critical size above which there will be no further reduction of strength with increasing sample size. Determination of the critical size requires testing of very large samples. Performing such tests is very difficult and costly and the number of tests on sufficiently large samples is very limited. The results of independent large scale tests carried out by Bieniawski (1968) on coal and by Pratt et al. (1972) on diorite suggest that the critical size for these materials is about 1 m.

In order to estimate the strength of large scale pillars, it is useful to find a relationship between the uniaxial compressive strength of intact rock and the pillar strength. This relationship bridges the size gap between the laboratory samples and large scale pillars. To this end, a reference pillar, representative of large scale pillars is defined. The size of the reference pillar must be no smaller than the critical size. Since the strength of the reference pillar is compared with strength of intact laboratory sample, the width to height ratio of the reference pillar must also be similar to that of laboratory specimen. Choosing the width of 1 m for the reference pillar ensures that the results will be valid for larger pillars (no size effect beyond the critical size of 1 m). In order to avoid the introduction of shape effects in this analysis, the width to height ratio of pillar is chosen as 0.5 to be similar to the diameter to length ratio of the laboratory test sample (ASTM, 2004).

Strength of the reference pillar ($W = 1$ m and $H = 2$ m) can be estimated from the empirical formulas. The ratio of the reference pillar strength to intact uniaxial compressive strength is defined as the in situ strength factor K. Using the empirical formulas in Table 1, in situ strength factor K ranges from 0.21 to 0.44 with the average value of 0.31. Based on comprehensive back analysis of pillars at the Quirke mine, Swan (1985) also found that the ratio of mean pillar strength to mean intact uniaxial compressive strength is 0.33 (Fig. 1).

2.2. Shape effect

The strength of pillars is also influenced by the shape of pillar expressed as the width to height ratio in the empirical formulas in Table 1. The relationship between the pillar strength normalized by intact uniaxial compressive strength and width to height ratio of the pillar is shown in Fig. 2. It can be observed that the pillar strength increases with increasing width to height ratio. This trend is similar to the one observed in the laboratory compression tests on specimens with different diameter to length ratios (Hawkes and Mellor, 1970). As in the case of laboratory tests, the actual factor which causes an increased strength for wider pillars is the end constraints which induce higher confinements within the pillar.

In order to explore the relationship between the width to height ratio and induced confinement, a series of uniaxial compression tests were carried out on elastic rib pillars using the finite difference code FLAC3D (Itasca Inc., 2009). Fig. 3 shows the profile of minor principal stress (confinement) normalized by the average major principal stress (average pillar stress) across the mid-height of pillars with different width to height ratios. It can be observed that increasing width to height ratio significantly increases induced confinement which in turn leads to higher pillar strength. It is also worth noting that for a pillar with W/H ratio of 0.5, tensile stresses are induced within the pillar. It suggests that the behavior of very slender pillars ($W/H < 0.5$) is governed by tensile mechanisms, splitting and buckling. Analyzing