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Reliability analysis of underground excavation in elastic-strain-softening rock mass

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ABSTRACT

This paper deals with the reliability analysis of a circular tunnel in elastic-strain-softening rock mass. Dilatancy angle which varies with softening parameter in different stress conditions is accounted for. Deterministic and probabilistic analyses of the circular tunnel in elastic-strain-softening rock mass are performed. Computational procedures for the first-order and second-order reliability methods (FORM/SORM) are used in the reliability analyses of the elastic-strain-softening model. The results are in good agreement with those from Monte Carlo simulations incorporating importance sampling. Reliability-based design of the required support pressure for the circular tunnel is efficiently conducted. The effect of positive correlation between compressive strength and elastic modulus of the rock mass on the reliability of the tunnel is discussed. The influence of *in situ* field stress and support pressure as random variables on the probability of failure of the tunnel is investigated.

1. Introduction

Convergence confinement method (CCM) is a widely used method in the design of support for underground excavations in rock masses. The method was initially developed in 1930s and refined by other researchers (e.g. Hoek and Brown, 1980; Brown et al., 1983; Hoek et al., 1995), as reviewed by Carranza-Torres and Fairhurst (2000). The method consists of three basic graphs: longitudinal deformation profile (LDP), support characteristic curve (SCC) and ground reaction curve (GRC). This research focuses on the GRC.

Analytical solutions are often not available due to the complexity of engineering conditions. Circular tunnels are special cases which have analytical solutions (Ogawa and Lo, 1987; Duncan-Fama, 1993; Wang, 1996; Carranza-Torres and Fairhurst, 1999; Sharan, 2003, 2008; Park et al., 2008) and can be used for preliminary analysis of underground excavations and to guide design.

It is more logical to regard the properties or input data of geomaterials as random variables rather than constant values because of their uncertainties. Reliability analyses that considered uncertainties in rock properties have been conducted by Mollon et al. (2009), Li and Low (2010), among others, using elastic-perfectly-plastic model. Further investigations are conducted in the present study involving more complex constitutive models, Lü and Low (2011), Lü et al. (2011, 2013) using elastic-perfectly-plastic model with first-order reliability method (FORM), second-order reliability method (SORM) or response surface method (RSM). Zeng and Jimenez (2014) applied a method using FORM and SORM to evaluate the reliability of series geotechnical systems (a layered soil and a circular rock tunnel in a Hoek-Brown rock mass). A method proposed by Zhao et al. (2014) using Least squares support vector machines (LS-SVM) based RSM combined with FORM is applied in tunnel reliability analysis of elastic-perfectly-plastic rock mass. Cho (2013) studied the reliability of a clayey soil slope considering multiple failure modes.

Hoek and Brown (1997) suggested elastic-brittle-plastic, elastic-strain-softening and elastic-perfectly-plastic behaviors for very good quality hard rock masses, average quality rock masses and very poor quality soft rock masses, respectively. In fact most studies focus on the elastic-perfectly-plastic constitutive model for the theoretical and numerical analysis convenience. However, many rock masses belong to the average classification that is elastic-strain-softening model.

Low et al. (2011) and Lü et al. (2013) discussed about the system reliability. Most studies at present mainly considered the unsatisfactory performance of the tunnel as individual failure modes. Due to the correlation of the failure modes and their sharing of common uncertain variables, the component failures are usually somewhat correlated and of different relative

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importance to the system failure. A yet more rational approach is to carry out a tunnel support design to comply with a target *system* probability of failure. Details can be found in Section 8 in this paper.

In this paper, A VBA procedure of the elastic-strain-softening constitutive model is first created in Microsoft Excel to perform the iterative process and the reliability analysis of a circular tunnel in elastic-strain-softening rock mass subjected to a hydrostatic *in situ* stress field is conducted. Reliability index and failure probability with respect to plastic-zone radius and radial displacement of the circular tunnel are calculated. Probability density functions of plastic-zone radius and radial displacement are obtained using cubic spline interpolation method. System reliability-based design of support pressure to achieve an overall target reliability index is carried out. Other factors influencing the reliability analysis and design of the circular tunnel, such as the support pressure, the *in situ* field stress and the positive correlation between the compressive strength and the elastic modulus, are also investigated.

2. Hasofer-Lind index and FORM algorithm based on an intuitive perspective

The matrix formulation of the Hasofer-Lind index for correlated normals is (e.g. Ditlevsen, 1981):

$$\beta = \min_{\mathbf{x}\in F}\sqrt{(\mathbf{x}-\boldsymbol{\mu})^T\mathbf{C}^{-1}(\mathbf{x}-\boldsymbol{\mu})} \tag{1}$$

where **x** is a vector representing the set of random variables x_i, $\boldsymbol{\mu}$ is the vector of mean values; **C** is the covariance matrix; and F is the failure domain. According to Eq. (1), the Hasofer-Lind index can be regarded as the minimum distance in units of directional standard deviations from the mean-value point of the random variables to the boundary of the limit state surface.

An equivalent formulation (Low and Tang, 1997, 2004) for Eq. (1) is:

$$\beta = \min_{\mathbf{x}\in F}\sqrt{\left[\frac{x_i-\mu_i}{\sigma_i}\right]^T\mathbf{R}^{-1}\left[\frac{x_i-\mu_i}{\sigma_i}\right]} \tag{2}$$

where **R** is the correlation matrix; and σ_i is the standard deviation of random variable x_i.

Eq. (2) was preferred by Low and Tang (1997) rather than Eq. (1) because the correlation matrix **R** is easier to set up than the covariance matrix **C**, and conveys the correlation structure more explicitly.

For correlated non-normals, Eq. (2) can be rewritten as (Low and Tang, 2004):

$$\beta = \min_{\mathbf{x}\in F}\sqrt{\left[\frac{x_i-\mu_i^N}{\sigma_i^N}\right]^T\mathbf{R}^{-1}\left[\frac{x_i-\mu_i^N}{\sigma_i^N}\right]} \tag{3}$$

where μ_i^N and σ_i^N are the equivalent normal mean and equivalent normal standard deviation of random variable x_i, respectively. The values of μ_i^N and σ_i^N can be computed using the Rackwitz and Fiessler (1978) two-parameter equivalent normal transformation:

$$\sigma^N = \frac{\phi\{\Phi^{-1}[F(x)]\}}{f(x)} \tag{4}$$

$$\mu^N = x - \sigma^N\cdot\Phi^{-1}[F(x)] \tag{5}$$

where x is the original non-normal variate, $\Phi^{-1}[\cdot]$ is the inverse of the standard normal cumulative distribution function (CDF), $F(x)$ is the original non-normal CDF evaluated at x, $\phi\{\cdot\}$ is the probability density functions (PDF) of the standard normal distribution, and $f(x)$ is the original non-normal probability density ordinate at x. For correlated non-normals, the ellipsoidal perspective (Fig. 1) and the constrained optimization approach still apply in the original coordinate system, except that the non-normal distributions are replaced by an equivalent normal hyper-ellipsoid, centered not at the original mean of the non-normal distributions, but at the equivalent normal mean μ^N. The extension of the Hasofer–Lind index to correlated non-normals is known as the first-order reliability method (FORM).

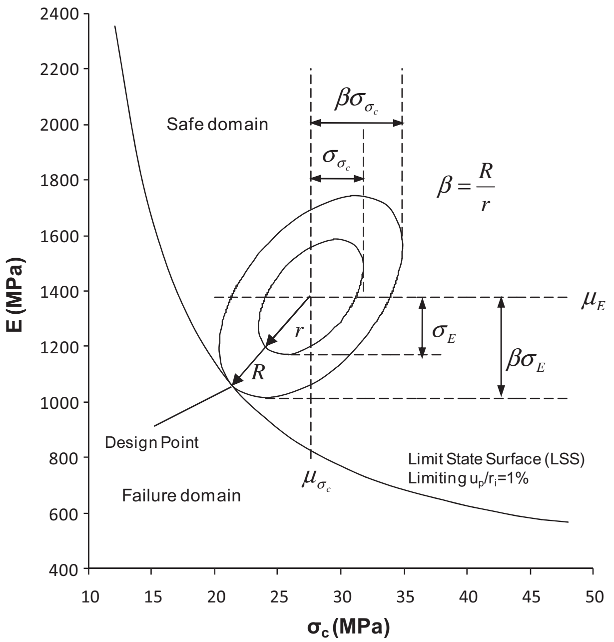

Fig. 1. Design point and normal dispersion ellipsoids illustrated in the original random variables' space.

Eq. (3) can be rewritten as follows (Low and Tang, 2007):

$$\beta = \min_{\mathbf{x}\in F}\sqrt{(\mathbf{n})^T\mathbf{R}^{-1}(\mathbf{n})} \tag{6}$$

where **n** is a column vector of n_i and $n_i = (x_i - \mu_i^N)/\sigma_i^N$.

When the value of n_i is varied (automatically) during constrained optimization, the corresponding value of x_i is automatically calculated as:

$$x_i = F^{-1}[\Phi(n_i)] \tag{7}$$

The Low and Tang (2007) algorithm for FORM calculates the reliability index of Eq. (6) using Microsoft Excel's built-in optimization routine *Solver*, subject to the constraint that the performance function $g(\mathbf{x}) = 0$ (where the **x** values are program-calculated from Eq. (7)), and by automatically changing the values of n_i.

Based on the reliability index, the probability of failure can be evaluated from:

$$p_f \approx 1 - \Phi(\beta) \tag{8}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal variable.

3. Problem description

In this paper first a VBA procedure is created and verified in Microsoft Excel to perform the iterative process of a circular tunnel initially subjected to a hydrostatic *in situ* stress and then reliability analysis based on this procedure is carried out. The problem is as follows.

When an underground opening is excavated in a stressed rock mass, the stresses in the vicinity of the new opening are