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Stability analysis of unlined elliptical tunnel using finite element upper-bound method with rigid translatory moving elements

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ABSTRACT

The stability of an unlined elliptical tunnel in cohesive-frictional soils is determined. The analysis has been performed with two methods: finite element upper-bound method with plastic deformation elements (UP-PDE) and finite element upper-bound method with rigid translatory moving elements (UP-RTME). UP-PDE has been used to study tunnel stability by many scholars. The UP-RTME in combination with a finite element approach and triangular rigid translator moving elements is presented in detail. In the proposed method, the node coordinates and velocities of rigid elements are treated as unknowns without considering the rotating freedom. A specific plane strain formulation is proposed using nonlinear programming, and the optimal slip lines are determined by automatically adjusting the velocity discontinuities. Solutions for the influence of a range of soil parameters, dimensionless depths C/D and dimensionless spans B/D on the stability numbers $\gamma D/c$ and collapse mechanisms are solved using this method. The $\gamma D/c$ values increase with ϕ and decrease with C/D and B/D. $\gamma D/c$ is less sensitive to C/D as ϕ increases. The collapse mechanisms of unlined elliptical tunnels comprising two groups of slip lines are also presented, and they explicitly reflect the relative movement of blocks. The results show that these two methods are in agreement with each other.

1. Introduction

The stability and collapse mechanisms of tunnels are important issues that have to be addressed in tunnel engineering. Studies in this regard have typically employed the limit analysis method, based on Drucker et al.'s (1951, 1952) plastic bounding theorems. The upper bound method of limit analysis is usually applied to materials that can be idealized as perfectly plastic with an associated plastic flow rule and small deformations; furthermore, it is necessary to construct a kinematically admissible velocity field in which the strain rates satisfy the plastic flow rule and velocities satisfy the boundary conditions.

Circular tunnels are particularly stable and are convenient to be excavated using a shield machine. Therefore, their stability has been investigated extensively. The rigid blocks upper bound method is commonly used to analyze stability problems with a postulated admissible collapse mechanism. For example, Atkinson and Potts (1977) discussed the limit force of shallow tunnels in cohesionless soil. Davis et al. (1980) studied the stability of shallow tunnels in clay ground based on four types of simplified collapse mechanisms. Osman et al. (2006) developed a continuous plastic deformation mechanism that includes a formula for predicting the ground deformation. Klar et al. (2007) substituted the plastic velocity field for the elastic displacement field to discuss the tunnel stability in clay ground. Sloan and Assadi (1993) were the first to apply finite element limit analysis to investigate the stability of a circular tunnel in cohesive soil whose shear strength varied linearly with depth. Lyamin and Sloan (2000) subsequently studied stability using a more efficient nonlinear programming technique. Sahoo and Kumar (2012, 2014) determined the stability of a long unsupported circular tunnel in the presence of pseudo static horizontal earthquake body forces. Yamamoto et al. (2011a, 2013) and, later, Sahoo and Kumar (2013a, 2013b) analyzed the stability of single and dual circular tunnels.

Compared to circular tunnels, square and rectangular tunnels are more difficult to construct due to poorer stability. Nonetheless, they are used in practice as they maximize the useable space while minimizing the amount of soil to be excavated. Some studies have focused on the stability of such tunnels as well. Assadi and Sloan (1991) studied the active and passive undrained failure of square tunnels. Sloan and Assadi (1991) and Wilson et al. (2013) discussed the stability of shallow square tunnels in soils whose undrained shear strength increases linearly with

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depth. Yang and Yang (2010) and, later, Yamamoto et al. (2010, 2011b) investigated the stability of square tunnels in cohesive-frictional soils. Abbo et al. (2013) studied the stability of rectangular tunnels whose width is larger than the height using the rigid block upper bound method.

In fact, except for the inner contour line of tunnels mentioned above, the value of the dimensionless span B/D for the tunnel, which has span B and height D, strongly influences the stability of tunnels. Abbo et al. (2013) analyzed the effect of the span on the stability of rectangular tunnels. However, rectangular tunnels are seldom used in tunnel engineering owing to irregularities in tunnel profile, which negatively impacts the structural stability. In mountain tunnel construction, multi-lane highway tunnels, whose widths are larger than the heights, and railway tunnels, whose heights are larger than the widths, are widely used. It is essential for civil engineers to focus on the stability of these tunnels and to study their collapse mechanisms. Generally, these tunnels contours comprise complex curves, and only few studies have investigated their stabilities using the upper bound method. To generalize these studies, it is reasonable to simplify complicated tunnels as unlined tunnels with an elliptical outline. Accordingly, this study aims to determine the stability of unlined elliptic tunnels in cohesive-frictional soils. The effects of soil properties, dimensionless depths, C/D and dimensionless spans B/D on the stability of unlined elliptic tunnels in a gravity field are investigated. Yang et al.'s (2014) finite element upper-bound method with rigid translatory moving elements (UP-RTME), presented in the following section, for solving optimization problems using nonlinear programming routines is applied to calculate the bounds and determine the evolution characteristics of the critical collapse mechanisms. To verify the solutions, tunnel stabilities were also studied using the finite element upper-bound method with plastic deformation elements (UP-PDE) presented by Sloan (1989, 1995).

2. Problem description

Fig. 1 shows the plane strain analysis model of an unlined elliptical tunnel for drained condition. As shown in Fig. 1, the ground is modeled as a uniform Mohr–Coulomb material with unit weight γ, internal friction angle, $\phi = \phi'$ and cohesion $c = c'$. ϕ' and c' denote the values of soil parameters associated with the effective stresses. The elliptical tunnel has a height D, a span B and a depth C. The stability of the tunnel is described conveniently by the dimensionless stability number, $\gamma D/c$ which is a function of, ϕ, c, C/D and, B/D as shown in the following equation:

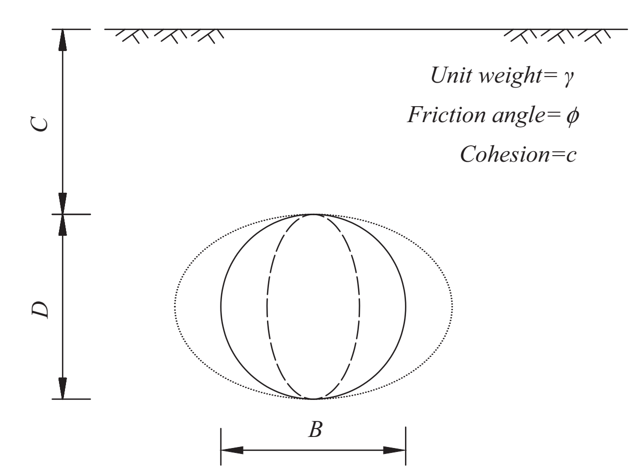

Fig. 1. Stability analysis model of unlined elliptical tunnel.

$$\gamma D/c = f(\phi, C/D, B/D) \tag{1}$$

As it has been assumed that no surcharge loading acts at the ground surface, in the above expression, γ is the maximum unit weight which can be borne by the unlined tunnel for given c, ϕ, C/D and B/D without any collapse.

The typical finite element meshes for an elliptical tunnel with $C/D = 1.5$ and $B/D = 1.0$ are shown in Fig. 2. Fig. 2(a) shows the 230 rigid elements and 327 velocity discontinuities that were considered to model the failure of the elliptical tunnel with UP-RTME. The finite element mesh shown in Fig. 2(b) is illustrative of that used for UP-PDE. The actual meshes adopted in the analysis were much more refined where 2884 elements and 4268 velocity discontinuities are used. Because the model is symmetric, only the right half part is studied. The boundary parameters L_1 and L_2 are 15 and 40 m, respectively. The bottom and right boundaries are constraint with $u = 0$, $v = 0$ and the horizontal velocity component in the left boundary is set as zero. No velocity constraints are imposed in the contour of the tunnel and along the ground surface. The model is discretized in the structured meshes, and a local mesh refinement method is presented for regions where failure may occur. Similar discrete methods of the models are used for other cases.

3. Finite element upper-bound method with rigid translatory moving elements (UP-RTME)

The accuracy of the rigid blocks upper bound method strongly depends on a postulated admissible collapse mechanism, which comprises rigid blocks with velocity discontinuities. Several studies have been conducted of this issue. Milani and Lourenco (2009) used rigid triangular elements with Bezier curved edges to build sequential linear programming models. Hambleton and Sloan (2013) presented a numerical technique for computing rigorous bounds on limit loads by optimizing rigid block mechanisms. Their methods were based on multiple successive perturbations, where the optimization problem corresponding to each perturbation step is solved using second-order cone programming. Yang et al. (2014) later established a nonlinear programming model using UP-RTME. The coordinates of the rigid triangular elements nodes, x_i and y_i are now treated as unknowns to be determined as part of the solution procedure. For models with a less elements and velocity discontinuities, the solving process is found to be simple, and the obtained critical collapse mechanisms explicitly reflect the relative movement of blocks.

3.1. Rigid translatory moving elements

As shown in Fig. 3, rigid translatory moving elements are triangular elements possessing the characteristics of translation and movable nodes. A velocity discontinuity occurs at the common edge between two adjacent elements, as defined by the nodal pairs ①② and ③④. (u_z, v_z) and (u_y, v_y) are the velocities in the x- and y-directions for the two adjacent elements, respectively. The node coordinates, (x_1, y_1) and, (x_2, y_2) are treated as unknowns in addition to the element velocities.

3.2. Constraints in velocity discontinuities

To be kinematically admissible, the normal and tangential velocity jumps (Δv, Δu) across the discontinuity must satisfy the flow rule, which, for a Mohr–Coulomb yield criterion, is of the form

$$\Delta v = |\Delta u| \tan\phi \tag{2}$$

In order to eliminate the absolute value, the parameters ξ_i' and ξ_i'' are introduced.