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Internal forces of underground structures from observed displacements

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ABSTRACT

This paper presents a method that provides a solution to the long standing problem of calculating internal force distributions based on displacement measurements of piles, retaining walls and tunnels. It is based on the principle of virtual work and therefore, analytically correct in the linear elastic range, and works without the need of any boundary conditions.

The validation against multiple case studies, showcasing loading conditions including seismic, earth pressures, external loads, or sliding slopes in multiple ground conditions and construction processes, confirms its flexibility and applicability to any structure where displacements are observed. Although the validation presented here applies to bending moments and axial forces, the method is theoretically correct and applicable to other internal force distributions.

1. Introduction

The behaviour and structural design of underground structures is governed by the distribution of internal forces. Out of these internal forces, bending moments are most critical for structures supporting bending forces, such as laterally loaded piles and retaining walls, and subsequently for the amount of reinforcement that the structure must be provided with. In tunnels, axial forces are equally relevant, not for reinforcement considerations only, but to guarantee its stability as well. However, despite the importance of these internal forces, traditional monitoring techniques of these structures concentrate on measuring total or relative deformations to verify design assumptions rather than enabling direct conclusions about the governing internal forces of the structure itself.

This disconnection between monitoring and design parameters arises for two main reasons (Fuentes, 2012): lack of proven and widely accepted monitoring techniques to measure internal forces, especially bending moments, and the lack of a general method to translate displacement measurements into internal forces.

With regards to bending moments, and in response to the first of the above shortcomings, some have recently developed techniques using fibre optics that are capable of measuring bending moments or curvature indirectly (e.g. see Inaudi et al., 1998; Mohamad et al., 2010, 2011, 2012; Fuentes, 2012). However, this technique is still suffering from the fact that measurements are indirect – i.e. curvature is inferred from axial strains – and that in order to obtain other relevant parameters, such as displacements, a cumbersome double integration needs to be carried out. Nip and Ng (2005) illustrated the problems of this integration process based on beam theory and overcame this successfully defining multiple boundary conditions over a controlled pile test and applying an iterative process to calculate the integration constants and fitting parameters. However, due to these conditions, the method cannot be simply used for other structures where less control over the boundary conditions is present. Mohamad et al. (2011) used a numerical integration and boundary conditions of zero rotation and displacement at the wall toe, which were reasonable due to the depth of the wall under consideration. For less deep structures this assumption would be incorrect and hence further measurements, additional known boundary conditions or both must be provided. Furthermore, it must be noted that calculation of displacements from curvature provides only part of the total displacement as it ignores rigid body translations and rotations.

The second shortcoming, translating displacements into bending moments or curvature, has been, to date, challenging. It involves the double derivation of a fitted curve to the displacement profile that, as Brown et al. (1994) highlighted, often presents difficulties and errors that propagate through the double derivation process. In order to reduce these errors, multiple readings are needed and other boundary conditions need to be imposed in advance so that the results are acceptable. Hence, although satisfactory solutions have been provided in the literature, these

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Nomenclature

a_j	distance from toe of the structure to the position where the unit-load is applied
A	area of cross section of the structure
AIC	Akaike Information Criterion
$\mathbf{B_N}$, $\mathbf{B_M}$, $\mathbf{B_V}$, $\mathbf{B_T}$	matrices which elements are the integrals resulting from the application of the method corresponding to the normal, moment, shear and torsion internal force distributions respectively
C_0, C_1, C_j, C_n	coefficients of linear equation representing the internal force distribution of the real structure
$\mathbf{C_N}$, $\mathbf{C_M}$, $\mathbf{C_V}$, $\mathbf{C_T}$	arrays of coefficients defining the normal, moment, shear and torsion internal force distributions respectively
$d\delta$, $d\theta$, $d\rho$, $d\gamma$	small displacement of the real structure
E	Young's modulus
G	shear modulus
I	second moment of inertia of cross section
I_p	polar moment of inertia
$f_0(x), f_1(x), f_j(x), f_n(x)$	functions of linear equation representing the internal force distribution of the real structure
f_n	function under evaluation
$\hat{f}$	estimate of function
h	embedded length in retaining walls
H	retained height in retaining walls
k	number of field measurements of displacements for the real structure
L	structure length in retaining walls/piles
n	indication on the number of functions used to approximate the internal force distributions
$M(\varphi)$	bending moment of the tunnel
$M_{(j)1}(x)$	bending moment in the pile/retaining wall caused by the unit-load force
$M_{(j)1}(\varphi)$	bending moment in the tunnel lining caused by the unit-load force
N_1, M_1, V_1 and T_1	normal stress, bending moments, shear stress and torsion internal force distributions of the unit-load structure
N_{real}, M_{real}, V_{real} and T_{real}	normal stress, bending moments, shear stress and torsion internal force distributions of the real structure
$q(x)$	external pressure acting on retaining walls/piles
$q(\varphi)$	external pressure acting on tunnel lining
R	radius of tunnel
SSE	Sum of Square of Errors
t	tunnel lining thickness
x	distance from the toe of the retaining wall/pile
u	displacement of real structure in retaining walls/piles
$\mathbf{u}$	array of field Observed displacements in retaining walls/piles
u_j	displacement of the real structure at the point j where the unit-load is applied
u_D	bending component of field measurement displacements in retaining walls/piles
u_D	lateral displacement of pile/retaining wall causing bending moments or radial component of distortion displacement at a point of the tunnel lining
u_C	uniform convergence displacement
u_O	observed lateral displacement of pile/retaining wall, radial displacement of the tunnel lining in the rotated tunnel
u_O^{SL}	radial displacement at the tunnel springline observed
u_{RB}	radial component of rigid body displacement at a point of the tunnel lining
u_{RB}^{SL}	radial component of rigid body displacement at the tunnel springline
u_D^{SL}	radial component of distortion displacement at the tunnel springline
u_O^{CR}	radial displacement at the tunnel crown observed
u_{RB}^{CR}	radial component of rigid body displacement at the tunnel crown
u_D^{CR}	radial component of rigid body displacement at the tunnel crown
u_{real}	field Observed displacements in retaining walls/piles
α_s	shear coefficient
β_j	angle measured from the vertical direction clockwise to the point of application of the unit-load
δ	translation displacement
φ	angle measured from the vertical direction at the tunnel crown and clockwise
ψ	rigid body rotation

apply to specific conditions and structures and therefore, need to be used with caution elsewhere.

The situation in tunnels is even more problematic as the available solutions to obtain bending moments and axial loads from displacements involve back-calculation and iterative processes using models that are successful in forward prediction – e.g. continuum models (Muir Wood, 1975; Curtis, 1976; Einstein and Schwartz, 1979; Duddeck and Erdmann, 1985; El Naggar et al., 2008; Carranza-Torres et al., 2013), convergence-confinment methods (e.g. Panet and Guenot, 1982), bedded beam springs (ITA, 1988; Oreste, 2003) or finite element analysis. Although satisfactory in its forward use, they also apply to specific conditions and still do not provide an independent check on the original calculation method.

This paper presents the first application of the unit-load to the calculation of internal forces – You et al. (2007) used its more typical application for displacement calculations for a shield tunnel and, similarly, Kim (1996) used it for validating the displacements obtained from predictive methods in model tunnels. It is based on the principle of virtual work, and enables calculating the internal force distributions of piles, retaining walls and tunnels when the displacements of the structure are known, without the need of any boundary conditions. The validation here concentrates on bending moments for all three structure types and axial forces in tunnels, as they are the most relevant to their performance. However, the methodology would equally apply to other internal force distributions.

2. The unit-load method in its traditional use

The unit-load (UL) method uses the principle of virtual work and is widely used in structural engineering for the calculation of displacements of structures. Its implementation involves the definition of two structural systems: one comprising the real structure with its external loads (denoted here as 'real') and the second (denoted as '1') consisting of the same structure with only a single unit-load applied at the point and in the direction of the displacement to be calculated. Once the two systems are defined, Gere and Timoshenko (1987) show that the displacement, u, of the real structure at the point of application of the unit-load is