Journal of Mathematical Psychology 84 (2018) 57–73

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

A system factorial technology analysis of the size congruity effect: Implications for numerical cognition and stochastic modeling

Daniel Fitousi [a,*], Daniel Algom [b]

[a] Department of Psychology, Ariel University, Israel
[b] Tel-Aviv University, Israel

HIGHLIGHTS

- The SCE is the phenomenon by which numerical magnitude interferes with or facilitates physical judgments of numerals.
- The SCE has been interpreted as a strong evidence for automatic activation of numerical magnitude.
- We have applied the system factorial technology (SFT, Townsend & Nozawa, 1995) – a stochastic modeling approach – to unravel the underlying mechanisms of the SCE.
- We found substantial evidence for serial minimum-time processing — an architecture that is inconsistent with automatic activation.
- We conclude that numerical information is not activated in an automatic fashion.
- We explain how the SCE can be present or absent depending on the specific model of serial minimum-time models.

ARTICLE INFO

Article history:
Received 25 October 2016
Received in revised form 5 March 2018
Available online 18 April 2018

Keywords:
Numerical cognition
Response times
Mental architecture
Stochastic modeling

ABSTRACT

We applied the methodology known as the system factorial technology (SFT) to diagnose the information-processing architecture underlying the size-congruity effect (SCE) in numerical cognition. The SCE documents the interference in judging the *physical* size of numerals when this size disagrees with their numerical magnitude or the facilitation when the two attributes agree. Traditional theories of the SCE implicate the automatic activation of numerical magnitude and hence the mandatory interaction in processing between number and size. In contrast, in a pair of experiments we found serial minimum-time processing of number and size, an outcome which excludes the possibility of interaction. In the face of this architecture, we still recorded appreciable amounts of redundancy gains when number and size corresponded (=SCE). However, we show that this SCE does not derive from an interaction in processing. We show that, given stochastic independence, certain species of serial self-terminating models actually mandate the SCE. Other species of serial self-terminating models do not allow an SCE, an outcome that accounts for the absence of an observable SCE in a fair number of studies. Our results are inconsistent with the belief that numerical information is activated in an automatic fashion under all circumstances.

In (pure) mathematics, numbers are dimension-less objects. In the empirical world, by contrast, numbers come dressed in physical dimensions. The numbers in everyday life are written on classroom blackboards, appear on computer screens, or sounded by a teacher. In the absence of a physical/sensory dimension, no human would have ever perceived a number. Given that each number is a combination of semantic and physical attributes, one can ask: How do people perceive the *physical* dimension of the number? In particular, does numerical magnitude affect perception of the digit's physical size? Many studies (see, Dehaene, 1997, for review) show that it does, which, in turn, led researchers to argue that the arithmetic/semantic dimension – numerical magnitude – is processed in an automatic fashion just whenever a numeral is presented for view for any purpose. Given the automatic processing of number, a number–physical size interaction is inevitable whenever different numbers in different physical sizes are processed (especially in tasks referring to physical size). In this study, we subjected the assumption of automatic number processing to new scrutiny, harnessing state-of-the-art tools of stochastic modeling. To anticipate our conclusions, we show that the architecture governing processing excludes the possibility of a number–size interaction. Consequently, we challenge the assumption of automatic processing of numerical magnitude.

* Corresponding author.
E-mail address: danielfi@ariel.ac.il (D. Fitousi).

https://doi.org/10.1016/j.jmp.2018.03.006

1. The size congruity effect

When presented with a pair of numerals that differ in physical size, people select the *physically* larger member of the pair faster when this member is also numerically larger than when it is numerically smaller. Thus, the selection of the larger format is faster in the pair, 7-2, than in the pair, 2-7. The dimensional values match in the first pair (a congruent display), but conflict in the second pair (an incongruent display). The difference in performance between congruent and incongruent trials (favoring the former) defines the size congruity effect (SCE). One does not need to present pairs of numerals in order to observe the SCE. When presented with a single numeral, it takes people longer to decide that the physical format is small when the numeral is 9 than when the numeral is 2 (Algom, Dekel, & Pansky, 1996; Choplin & Logan, 2005; Fitousi, 2014). The SCE has been obtained with numbers written in different notations and in different tasks (e.g., Besner & Coltheart, 1979; Fitousi, 2010; Ito & Hatta, 2003; Schwarz & Ischebeck, 2003), with natural and negative numbers (Henik & Tzelgov, 1982; Pansky & Algom, 1999, 2002; Pinhas & Tzelgov, 2012; Pinhas, Tzelgov, & Guata-Yaakobi, 2010; Schwarz & Heinze, 1998; Szücs & Soltész, 2008; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009; Tzelgov, Meyer, & Henik, 1992), and with single- and double-digit numbers (Fitousi & Algom, 2006; Fitousi, Shaki, & Algom, 2009; Ganor-Stern, Tzelgov, & Ellenbogen, 2007; Santens & Verguts, 2011). Given its pervasiveness, the SCE is commonly taken to reflect on the mandatory processing of numerical magnitude. It is this view that we challenge in the current study.

2. Is the SCE mandatory?

The impressive evidence notwithstanding, pervasive biases in the standard experimental design call into question the obligatory nature of the SCE. Little stimulus alchemy suffices to eliminate the SCE or reverse the effect such that physical size intrudes on perception of numerical magnitude more than vice versa (the reverse-SCE; Fitousi & Algom, 2006). Algom et al. (1996) identified two critical biases prevalent in published SCE studies. First, there is a glaring asymmetry in the number of stimuli used for the numerical and the physical dimensions. Typically, the numbers 1 to 9 (inclusive) are used for the former, but only two or three values (small, medium, large) are used for the latter. Virtually all pertinent research pitted a finely grained numerical dimension against a coarse physical dimension. This asymmetry itself can determine the observed interaction (=SCE). Melara and Mounts (1994) have shown that the mere number of stimuli on an irrelevant dimension affects classification performance on the relevant dimension (see also Melara & Algom, 2003; Sabri, Melara, & Algom, 2001).

For another bias, the relative salience or discriminability of values along the number and the size dimensions was not matched. Discriminability is matched if the values along the numerical dimension differ perceptually from one another to the same extent as do values of size along the physical dimension. This information is critical because the more discriminable dimension can disrupt performance on the less discriminable dimension (e.g., Algom et al., 1996; Fitousi & Algom, 2006; Melara & Mounts, 1994). Thus, irrelevant numerical value will disrupt performance with physical size (=SCE) if the values of number differ psychologically from one another more than do the values of physical size from one another. Mismatched discriminability favoring numbers was present in virtually all studies of the SCE. However, when care was taken to match discriminability (and number of values along the two dimensions), the SCE collapsed. And, when physical size was purposely made more salient than numerical value, a reverse SCE emerged (Algom et al., 1996; Fitousi, 2014; Fitousi & Algom, 2006; Fitousi et al., 2009; Pansky & Algom, 1999, 2002; see also, Melara & Algom, 2003; Sabri et al., 2001). In Fig. 1, we illustrate the effect on the SCE of relative dimensional salience.

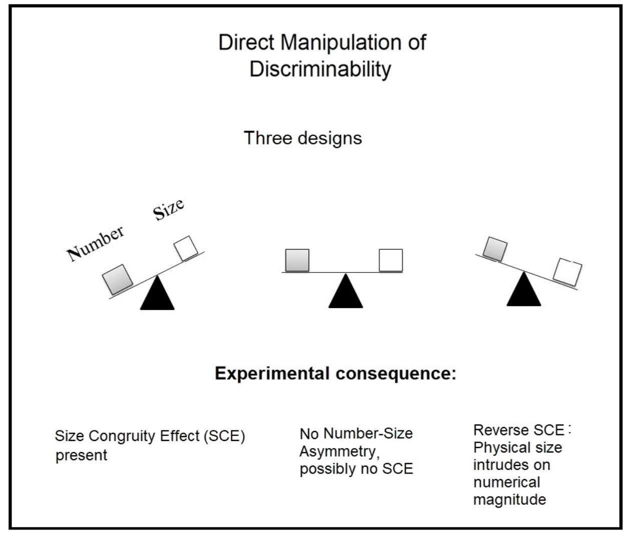

Fig. 1. Schematics of the influence of relative dimensional salience on the outcome of the SCE experiment. Left-hand panel: The numbers (N) are more discriminable than the values of physical size (S), the default setup in many studies. As a result, the irrelevant numbers intrude on the judgments of size, thereby generating the SCE. Middle panel: Numerical magnitude and physical size are matched in discriminability, resulting in the elimination of the SCE asymmetry in interference favoring numbers. Right-hand panel: Physical size is more discriminable than numerical magnitude, so that judgments of numbers are now subject to interference from physical size more than vice-versa (=reverse SCE). We uncover through SFT-guided collection of data and modeling the architecture underlying each of the outcomes depicted in Fig. 1 (see Discussion).

The malleability of the SCE casts doubt on the automatic nature of processing numerical magnitude. The question of the mandatory nature of the SCE is related to the fundamental contrast in cognitive science drawn by Garner (1970, 1974) between integral and separable dimensions (Garner, 1970, 1974, 1976; Garner & Felfoldy, 1970; see Algom & Fitousi, 2016, for a review). Separable dimensions of an object do not intrude on one another in processing, whereas integral dimensions do — due to holistic perception of the object. The elimination of the SCE in the Algom studies is consistent with the view that numerical magnitude and physical size of numbers are separable dimensions, whereas the robust appearance of the SCE is consistent with the traditional view that number and size are integral dimensions.

3. Are dimensions of number integral or separable?

All objects of perception are not experienced in the same way. Some show a character of totality, a *Gestalt* (cf. Heidbreder, 1933, p. 331), whereas other objects are experienced as separate features, which are simply grouped together in space and time. The objects in the first class are composed of integral dimensions, whereas the objects in the second class are composed of separable dimensions. Integral dimensions are processed in a unitary fashion, so that observers cannot attend exclusively to one dimension of the object without concurrently noticing the other dimension. It is the crosstalk between the constituent dimensions that yields the observed interaction in performance. Prototypical examples of integral dimensions are hue, brightness, and saturations of colors. According to the traditional view (Dehaene, 1997), numerical magnitude and physical size form another example of integral dimensions, expressed by the SCE.

Separable dimensions, by contrast, remain distinct in processing. Observers can successfully ignore a task-irrelevant dimension