



Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmpQuantum effect logic in cognition[☆]

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HIGHLIGHTS

- Illustration of novel quantum effect logic in quantum cognition.
- Explanation of strengths of effect logic w.r.t. traditional projection-based logic.
- Demonstration of the (Python / EfProb) tool support of the effect logic.
- Re-description of standard quantum cognition examples with effect logic.

ARTICLE INFO

Article history:

Received 16 February 2017

Received in revised form 27 June 2017

Available online xxxx

ABSTRACT

This paper illustrates applications of a new, modern version of quantum logic in quantum cognition. The new logic uses ‘effects’ as predicates, instead of the more restricted interpretation of predicates as projections – which is used so far in this area. Effect logic involves states and predicates, validity and conditioning, and also state and predicate transformation via channels. The main aim of this paper is to demonstrate the usefulness of this effect logic in quantum cognition, via many high-level reformulations of standard examples. The usefulness of the logic is greatly increased by its implementation in the programming language *Python*.

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1. Introduction

Quantum physics is ‘strange’ and full of counter-intuitive phenomena. Accordingly, the mathematical foundations of quantum physics are ‘strange’, with observations having a side-effect, non-commutativity, and context-dependence. In the relatively new area of quantum cognition one tries to model aspects of human cognition using the mathematical formalism of quantum theory. Indeed, some judgements and choices that people make are also ‘strange’, see for instance the many illustrations in [Kahneman \(2011\)](#). The idea is that quantum logic provides more suitable ‘laws of thought’ than classical Boolean logic ([Boole, 1854](#)), leading to more realistic predictions about the type of errors that humans make.

We briefly mention some of the relevant connections.

- The ‘conjunction and disjunction errors’ in for instance the Linda example from [Tversky and Kahneman \(1983\)](#) can be described logically via the fact that quantum conjunction &

and the associated disjunction, are not monotone in their first arguments.

- The relevance of the order of processing information is reflected by the non-commutativity of quantum conjunction &.
- The order effect in inference and priming is mirrored by the non-commutativity of conditioning of quantum states.
- Different perspectives can be described via the context-dependence in quantum theory, given by a choice of basis in a vector (Hilbert) space.
- Answering a question creates a cognitive context in a way that can be compared to how measurement in quantum theory changes the state of an object under measurement.

This non-commutativity and context-dependence is the main reason for moving to quantum logic and probability in cognition research, see for instance the books by [Bussemeyer and Bruza \(2012\)](#), [Khrennikov \(2010\)](#) for more information, references and discussion.

The main point of this paper is simple: if one decides to use quantum logic in cognition, then one might as well use the modern and expressive logic of effects, instead of the (restricted) logic of projections that is being used so far.

Historically, quantum logic started in the 1930s with [Birkhoff and von Neumann \(1936\)](#). They used a geometric interpretation

[☆] The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007–2013)/ ERC grant agreement nr. 320571.

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of logical predicates as closed linear subspaces of Hilbert spaces. These subspaces have the algebraic structure of an orthomodular lattice (Kalmbach, 1983). Such subspaces can equivalently be described as projection operations on Hilbert spaces. In 1990s these projections have been generalised to ‘effects’, that is, to positive operation below the identity, which are used as interpretations of predicates, see Foulis and Bennett (1994) (and independently Chovanec and Kôpka (2007); Giuntini and Greuling (1994), see Dvurečenskij and Pulmannová (2000) for an overview). Such effects have the mathematical structure of an effect algebra/module and form the basis of a novel approach to quantum probability (Gudder, 2007). In order to emphasise the difference, projections are sometimes called ‘sharp’ predicates, and effects are the ‘non-sharp’ ones. Similarly, in classical (non-quantum) probabilistic logic one can call $\{0, 1\}$ -valued functions sharp predicates; they are contrasted with $[0, 1]$ -valued functions, forming the non-sharp, fuzzy predicates. A crucial difference is that non-sharp predicates are closed under sequential conjunction $\&$, under scalar multiplication, and under predicate transformation, as will be explained below.

In a further line of development it has been shown that effect module structure arises naturally from a certain elementary categorical structure, called an ‘effectus’, see Cho, Jacobs, Westerbaan, and Westerbaan (2015) and Jacobs (2015). These effectuses form a general framework, including Boolean logic, classical probabilistic logic, and quantum logic. The quantum case involves non-commutativity of conjunction $\&$ and predicates that can have side-effects. The properly quantum effectuses form the abstract background for the current article. The main mathematical example is given by von Neumann algebras. Here we shall concentrate on more concrete examples, namely finite-dimensional Hilbert spaces—whose collections of operators form von Neumann algebras.

The main claim underlying this paper is that the quantum effect logic that started in the 1990s is more suitable for application in cognitive psychology, in comparison to the quantum subspace logic from the 1930s. This claim will be substantiated via several examples. These examples are not new, and in fact completely standard, taken from the well-known textbook (Busemeyer & Bruza, 2012). This is a deliberate choice, so that (cognition) researchers who are already familiar with these examples can now recognise them in the new effect formalism. Hence this paper does not offer any new – mathematical or psychological – results or theories; instead, it offers a broader logical perspective, supported by an implementation for calculating probabilities.

Indeed, the abstract logic and probability theory of effectuses come with an implementation in the popular programming language *Python*, called *EffProb*, for ‘Effectus Probability’.¹ This implementation defines primitives for probability and logic which can be used to define states, predicates and channels, and to compute validities and conditioned states. Interestingly, *EffProb* has a uniform approach to discrete, continuous, and quantum probability. Here we shall only use the quantum fragment of the language: all the calculations in the examples below are obtained via *EffProb*. How this is done in *Python* is illustrated in Appendix, for two of the examples. The *EffProb* formalisation of all the examples is made available, so that interested readers can see the details, make variations, and adapt to their own setting.

We should emphasise that we use the word ‘logic’ as in Birkhoff and von Neumann (1936), referring to a mathematical interpretation, and not to a formal system of rules of inference. So far, such a formal, symbolic logic of effects does not exist. Nevertheless, we

consider predicates as effects on a Hilbert space, closed under a number of logical operations, including notably conjunction and predicate transformation. Similarly, states are density matrices on a Hilbert space, closed under certain operations, including conditioning and state transformation. In the current approach we can form composite predicates, via these logical operations, for expressing different statements, such as “feminist & bank-teller” or “feminist | bank-teller” in the Linda example. Such complex predicates can be used to represent the situation at hand in logical form. Subsequently we can calculate the validity of such composite predicates.

In the remainder of this paper we describe in a step-by-step manner the main ingredients of the logic and probability theory of effectuses. Illustrations play an important role and are provided along the way. The reader is assumed to be familiar with the basics of the Hilbert space formalism of quantum theory, see e.g. Nielsen and Chuang (2000), Rieffel and Polak (2011) or Busemeyer and Bruza (2012) and Yearsley and Busemeyer (2016). The exposition starts with the basic notions of state and predicate, and with the validity of a predicate in a state, expressed as probability, in the unit interval $[0, 1]$. This can be used immediately in the famous Linda example. After this illustration, some more background information about effects is given in Section 3, including a brief comparison to classical probabilistic logic using fuzzy predicates. Section 4 introduces conditioning (update, revision) of a state by a predicate, and illustrates this construct in the well-known polarisation example from physics. Subsequently, Section 5 describes channels, and forward state transformation and backward predicate transformation along a channel. These transformations are illustrated in an example involving a man and a woman with different perspectives on car brands. Finally, Section 6 illustrates how the relevance of the order of conditioning quantum states can be used to describe rulings in a court case depending on the order in which evidence is presented.

2. States, predicates and validity

This section briefly introduces states and predicates, and illustrates how they are used in the Linda example.

States and predicates form the basic ingredients of our framework. It is important to distinguish them clearly, since they play completely different roles and have different operations. For instance, predicates are closed under multiplication with a scalar from the unit interval $[0, 1]$, but states are not. Intuitively, a state ω captures a certain state of affairs, and a predicate p captures a property of that state. We shall write $\omega \models p$ for validity: the probability, expressed as a number in the unit interval $[0, 1]$, that property p holds in the state ω . This validity $\omega \models p$ may also be read as the expected value of property p in state ω .

Let $\mathcal{H}$ be a finite-dimensional complex Hilbert space. A *state* ω of $\mathcal{H}$ is a positive operator on $\mathcal{H}$ with trace one. That is, ω is linear function $\omega : \mathcal{H} \rightarrow \mathcal{H}$ satisfying $\omega \geq 0$ and $\text{tr}(\omega) = 1$. A state is often called a *density matrix*. The canonical way to define a state is to start from a vector $|v\rangle \in \mathcal{H}$ with norm 1, and consider the operator $|v\rangle\langle v| : \mathcal{H} \rightarrow \mathcal{H}$. It sends an arbitrary element $|w\rangle \in \mathcal{H}$ to the vector $|v\rangle\langle v|w\rangle$. An arbitrary state is a convex combination of such vector states $|v\rangle\langle v|$.

A *predicate*, also called an *effect*, is a positive operator p on $\mathcal{H}$ below the identity: $0 \leq p \leq \text{id}$. The identity id is given by the identity/unit matrix, and corresponds to the *truth* predicate, often written as **1**. For each predicate p there is an orthosupplement, written as $p^\perp$ or as $\sim p$, playing the role of negation. It is defined by $\sim p = \text{id} - p$, and satisfies: $\sim\sim p = p$ and $p + \sim p = \mathbf{1}$. Section 3 gives more mathematical background information about predicates; at this stage we concentrate on how they are used.

¹ *EffProb* is publicly available via the website efprob.cs.ru.nl, together with an extensive user manual. Kenta Cho and the current author are the main developers, see Cho and Jacobs (2017) for an overview.

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