Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Quantum field inspired model of decision making: Asymptotic stabilization of belief state via interaction with surrounding mental environment

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HIGHLIGHTS

- A quantum-like model of the process of decision making is presented.
- The process of decision making is modeled as interaction with mental reservoir.
- Applications to modeling of voters' behavior and consumer's persuasion are presented.

ARTICLE INFO

Article history:
Received 7 June 2017
Received in revised form 14 October 2017
Available online xxxx

Keywords:
Decision making
Quantum-like model
Mental (information) environment
Open quantum systems
Dissipation of uncertainty
Voters' behavior
Consumers' persuasion

ABSTRACT

This paper is devoted to a justification of quantum-like models of the process of decision making based on the theory of open quantum systems, i.e. decision making is considered as decoherence. This process is modeled as interaction of a decision maker, Alice, with a mental (information) environment $\mathcal{R}$ surrounding her. Such an interaction generates "dissipation of uncertainty" from Alice's belief-state $\rho(t)$ into $\mathcal{R}$ and asymptotic stabilization of $\rho(t)$ to a steady belief-state. The latter is treated as the decision state. Mathematically the problem under study is about finding constraints on $\mathcal{R}$ guaranteeing such stabilization. We found a partial solution of this problem (in the form of sufficient conditions). We present the corresponding decision making analysis for one class of mental environments, the so-called "almost homogeneous environments", with the illustrative examples: (a) behavior of electorate interacting with the mass-media "reservoir"; (b) consumers' persuasion. We also comment on other classes of mental environments.

1. Introduction

The recent years were characterized by explosion of interest in applications of the mathematical formalism of quantum theory to studies in cognition, decision making, psychology, economics, finance, and biology, see, e.g., the monographs (Asano, Khrennikov, Ohya, Tanaka, & Yamato, 2015; Bagarello, 2012; Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; Khrennikov, 2004a, 2010) and a few representative papers (Asano, Basieva, Khrennikov, Ohya, Tanaka, & Yamato, 2013; Asano, Ohya, Tanaka, Basieva, & Khrennikov, 2012; Bagarello, 2015a, b; Bagarello & Gargano, 2017; Bagarello & Haven, 2016; Bagarello & Oliveri, 2010; Busemeyer, Pothos, Franco, & Trueblood, 2011; Busemeyer, Wang, Khrennikov, Basieva, &, 2014; Busemeyer, Wang, & Townsend, 2006; Danilov, Lambert-Mogiliansky & Vassili Vergopoulos, 2016; Denolf & Lambert-Mogiliansky, 2016; Dzhafarov, 2014; Dzhafarov & Kujala, 2012; Haven & Khrennikov, 2016; Hawkins & Frieden, 2012, 2017; Khrennikov, 2004b, 2006, 2016, 2017; Khrennikov & Basieva, 2014a, b; Khrennikov, Basieva, Dzhafarov, & Busemeyer, 2014; Kvam, Busemeyer, & Lambert-Mogiliansky, 2014; Lambert-Mogiliansky, 2017; Plotnitsky, 2014; Pothos & Busemeyer, 2009, 2013) (the first steps in this direction were done long time ago, see, e.g., Khrennikov, 1999).

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https://doi.org/10.1016/j.jmp.2017.10.002
0022-2496/

The approach explored in such mathematical modeling is known as *quantum-like.* In this approach an agent (human, animal, or even cell) is considered as a *black box* processing information in accordance with the laws of quantum information and probability theories. Thus the quantum-like modeling is basically quantum informational modeling (although this characteristic feature is typically not emphasized, cf., however, with Asano, Basieva, Khrennikov, Ohya, Tanaka, & Yamato, 2015).[1]

Quantum-like models have to be sharply distinguished from genuinely quantum physical models of cognition which are based on consideration of quantum physical processes in the brain, cf. with Hameroff (1994) and Penrose (1989). Although the quantum physical models have been criticized for mismatching between the temperature and space–times scales of the quantum physical processes and neuronal processing in the brain, see especially Tegmark (2000), they cannot be rejected completely and one may expect that quantum-like models of cognition will be (soon or later) coupled with real physical processes in the brain, see Busemeyer, Fakhari, and Kvam (in press), de Barros (2012), de Barros and Suppes (2009), Khrennikov (2011), Melkikh (2013, 2014) and Takahashi (2014) for some steps in this direction.

The quantum-like approach generated a variety of models of cognition and decision making. In the simplest model (Khrennikov, 1999, 2004a), the mental state (the belief state) of an agent, Alice, is represented as a quantum state ψ and questions or tasks as quantum observables (Hermitian operators). Answers to the questions are given with probabilities as determined by Born's rule. For Hermitian operator A with the purely discrete spectrum, Born's rule can be written as

$$p(A=\alpha_k)=\|P_{\alpha_k}\psi\|^2=\langle P_{\alpha_k}\psi,\psi\rangle, \tag{1.1}$$

where α_k is an eigenvalue of A and P_{α_k} is the projector on the eigenspace corresponding to this eigenvalue.

This model does not describe dynamics of the belief state in the process of decision making. Consideration of dynamics was introduced in the works of Khrennikov (2004b, 2006), and Pothos and Busemeyer (2009). In their dynamical model as in the previous models, an observable A corresponding to a question (task) faced by Alice is represented as a Hermitian operator. Then Hamiltonian H generating unitary dynamics

$$\psi(t)=U(t)\psi_0,\ \ U(t)=e^{-itH} \tag{1.2}$$

of the initial belief state ψ_0 is introduced, and Alice's decision is represented as measurement of the observable A at some instant of time. The authors of Pothos and Busemeyer (2009) presented cognitive arguments supported by experimental studies to determine the instant t_m of measurement. Here the probability of a particular answer is also determined by the Born rule, but applied to belief state $\psi(t_m)$. Of course, this is an important issue, since different values ot t_m can give rise to completely different results. In spite of the partial progress in determination of t_m in the article of Pothos and Busemeyer (2009), this complex problem cannot be considered completely solved. Moreover, in solving this problem Pothos and Busemeyer had to go beyond the quantum theory and to appeal to psychological theoretical and experimental studies. It would be attractive to solve this problem entirely in the quantum framework.

We remark that the Schrödinger equation describes the dynamics of an isolated system. In the presence of an environment, the dynamics of the system is non-unitary. Approximately (under some sufficiently natural conditions) it is described by the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation (often called simply the Lindblad equation), the simplest version of the quantum master equation. One of the main distinguishing features of such dynamics is that it does not preserve the pure state structure: it (immediately) transforms a pure initial state ψ_0 into a mixed state given by the density operator:

$$\rho(t)=U(t)\rho_0,\ \ U(t)=e^{-itL}, \tag{1.3}$$

where L is the generator of the GKSL-evolution and $\rho_0=|\psi_0\rangle\langle\psi_0|$. This dynamics is in general non-unitary. This equation describes the process of *the system adaptation to the surrounding environment.* This is the complex dynamical process combining the internal state dynamics of the system with adaptation to signals received from the environment. If the dynamics is discrete with respect to time, then it can be represented as *a chain of unitary evolutions and (generalized) quantum Bayesian updates.*[2] In this paper we cannot discuss this interesting issue in more detail, see Asano et al. (2013) for detailed consideration of a two dimensional example with application to the evolution theory.

In fact, GKSL-dynamics does not contradict the Schrödinger equation structure of the quantum evolution. Let us denote the system under study by S and the surrounding environment by $\mathcal{R}$ ("reservoir" for S.) Suppose that initial state of the compound system $S+\mathcal{R}$ is pure and separable. Dynamics in the state space of $S+\mathcal{R}$ is still unitary and given by a Hamiltonian for $S+\mathcal{R}$. The main distinguishing feature of this unitary dynamics is that (in the presence of interaction between S and $\mathcal{R}$) it induces *entanglement* and the state of the compound system becomes not more separable. Hamiltonians for the composite system $S+\mathcal{R}$ are very complex, since they include, in general, an infinite number of degrees of freedom of $\mathcal{R}$. Typically it is impossible to solve the Schrödinger equation for the state of composite system $S+\mathcal{R}$. (Although in some special cases, as those considered in this paper and others discussed in Bagarello (2012) analytic solutions can be found.) Therefore, most studies are restricted to the dynamics of the state $\rho(t)$ of S alone, which is described (approximately) by the GKSL-equation. But even if one were able to solve the Schrödinger equation for $S+\mathcal{R}$, the solution would be a very complex infinite-dimensional state vector. Since we are interested in behavior of S, we would then take the trace with respect to all degrees of freedom of $\mathcal{R}$ and obtain the state of S. A simple mathematical theorem implies that in presence of entanglement this trace-state cannot be pure, i.e., the state is described by density operator. Its dynamics under the GKSL-equation is known as *decoherence:* decreasing of state's purity (or coherence) in the process of interaction with an environment.

Since consideration of an isolated cognitive system is even a higher degree idealization than consideration of an isolated physical system, it is natural to modify the dynamical scheme of decision making based on unitary Schrödinger dynamics (Khrennikov, 2004b, 2006; Pothos & Busemeyer, 2009) and consider general dynamics of the belief-state, either by using the approximative GKSL-dynamics or by tracing the state of the compound system. Roughly speaking there is no choice: either one has to ignore the presence of environment or consider non-unitary dynamics, e.g., (1.3). Of course, such non-unitary dynamical model of decision making is much more mathematically complicated. However, it

[1] See, e.g. D'Ariano (2011) and Plotnitsky (2012) for the information approach to quantum mechanics.

[2] As was quickly understood in quantum physics, the Lüders projection postulate describes only one very special class of the quantum state updates resulting from measurements. We remark that already von Neumann accepted applicability of this straightforward form of the state update only for observables with non-degenerate spectra. Generally, in the case of an observable with degenerate spectrum, a pure pre-measurement state can be transferred into a mixed state (von Neumann, 1955). Later these considerations of von Neumann were elaborated in the form of the *theory of quantum instruments*, see Basieva and Khrennikov (2017) for non-physicist friendly presentation. The most consistent justification of this theory is obtained in the framework of the *theory of open quantum systems*.

Please cite this article in press as: Bagarello, F., et al., Quantum field inspired model of decision making: Asymptotic stabilization of belief state via interaction with surrounding mental environment. Journal of Mathematical Psychology (2017), https://doi.org/10.1016/j.jmp.2017.10.002.