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Multidimensional signal detection decision models of the uncertainty task: Application to face perception

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HIGHLIGHTS

- Extends multidimensional signal detection theory to the uncertainty paradigm.
- Clarifies misconceptions from prior applications of the uncertainty paradigm in vision research.
- Applies results to a face perception experiment replicating earlier findings but with realistic faces.

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ABSTRACT

The uncertainty paradigm has been used in vision research to evaluate whether stimulus components are processed independently or not. The paradigm consists of several experimental conditions from which sensitivity indices are estimated and combined to provide evidence for or against the independence of stimulus components in perception. In typical applications, a multicomponent stimulus differs in one of its components from a standard value and the observer needs to decide if the change is an increment or decrement. In the *certainty* condition, the observer knows which component will contain the change; in the *uncertainty* condition, the component that differs from standard is unknown. Performance across the two conditions can be compared to that which is predicted by independence of components. The mathematical foundations upon which performance indices are related to component independence have been inadequately examined in previous applications and we clarify many of these concepts here. We derive predictions for observer sensitivity in the *uncertainty* condition and a relative measure, root-mean-square (RMS) that incorporates both *uncertainty* and *certainty* performance for three major decision models using a signal detection theory framework: a distance-classifier, the optimal decision model, and a decisionally separable ("independent" decisions) strategy. We also consider, using these decision models, implications for sensitivity and RMS when stimulus components are perceptually correlated. We present data from an experiment involving the perception of facial features in order to demonstrate how to apply the theoretical results.

The uncertainty paradigm has been offered as a means of empirically verifying the independence of processing of stimulus components in a discrimination task (e.g., MacMillan & Creelman, 2005; Magnussen & Greenlee, 1997; Pelli, 1985; Thomas, Magnussen, & Greenlee, 2000; Thomas & Olzak, 1996; Wickens, 2002). The canonical structure of the task, as it pertains to multi-component independence, uses a two-component stimulus, such as two superimposed sine-wave gratings of differing orientations (Thomas & Olzak, 1996) or an oval of fixed height and luminance with the "components" of color (hue) and shape (width, Weerda, Vallines, Thomas, Rutschmann, & Greenlee, 2006). Fig. 1 diagrams a perceptual representation of the task as it is studied here. Each component can differ from an implicit (Morgan, Watamaniuk, & McKee, 2000) standard along a dimension of interest such as spatial frequency in the case of the gratings, or in color or shape in the case of

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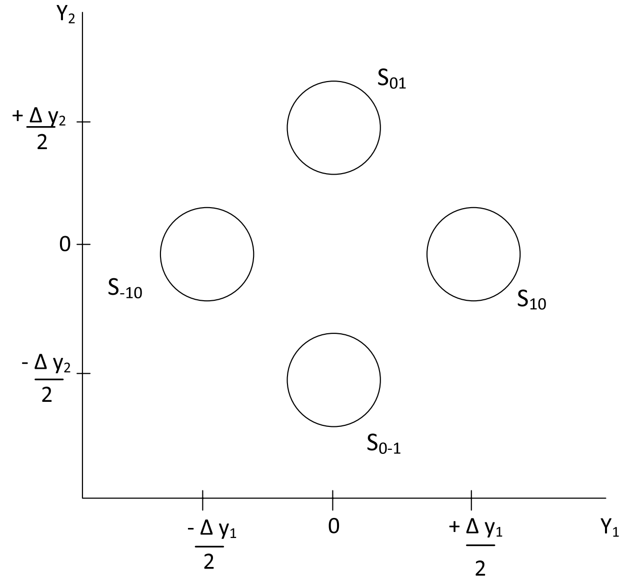

Fig. 1. Perceptual distributions of the two component stimulus set for the uncertainty paradigm. On a trial, if S_{10} or S_{01} is presented, the observer is to respond "increment", otherwise he or she is to respond "decrement". The observer bases the decision on the percept $(y_1,\ y_2)$ which is a sample from the presented stimulus distribution.

ovals either by an increment (relative to the standard) or a decrement. At least two conditions are needed. In the *certainty* condition, the observer knows in advance which of the two stimulus components will contain the change. In the *uncertainty* condition, the observer does not know which stimulus component will contain the difference from standard and must monitor information from both components. The observer's task is to indicate, when given a single stimulus on a trial, whether there was an increment in a component or a decrement. It has been asserted that, if the two components are processed by separate and statistically independent mechanisms, then performance must drop in the *uncertainty* condition relative to *certainty* (e.g., Thomas & Olzak, 1996) due to the combining of sources of noise. This is generally true; the specific amount that can be expected, however, depends crucially on the decision process the observer uses and the variance properties of the perceptual distributions of the stimulus components as this paper will illustrate. There has been some confusion regarding the exact amount of performance decrement to expect in this paradigm stemming from an underspecified or inaccurate analysis of the decision process (see e.g., Appendix of Thomas & Olzak, 1996). As has been observed by Ashby and his colleagues (Ashby & Gott, 1988; Ashby & Perrin, 1988; Ashby & Townsend, 1986; Maddox, 1992; Silbert & Thomas, 2013; Thomas, 1995, 1996, 1999, 2003), a complete understanding of perceptual mechanisms requires a full specification of the decision process the observer may use in the given task.

We begin by defining two important concepts of the signal detection framework that are used in the representation of this task: the empirically computed sensitivity index, $d'_{\text{empirical}}$, and a relative performance metric frequently used in applications (root-mean-square, RMS) which compares performance in the *uncertainty* condition to that of the *certainty* condition. We then present three major decision models (a distance-classifier, optimal responding, and a decisionally-separable strategy) together with their predictions for sensitivity ($d'_{\text{empirical}}$) in the *certainty* and *uncertainty* conditions and RMS under varying relative component variances beginning with the independent components case. Following this, we extend the analysis to correlated components. Finally, we demonstrate how to apply the results of the theoretical analysis in the context of an experiment investigating the perception of facial features when these should lead to interactions versus when they should be independently processed. Throughout the theoretical presentation, we strive to maintain detail and clarity in articulating the assumptions of representation and decision to avoid further misspecifications that may produce erroneous performance predictions.

1. The two component uncertainty task: notation, representation, and assumptions

As seen in Fig. 1, in the canonical version of this paradigm, a set of four stimuli, each denoted by S_{ij} are constructed from two components, Y_1 and Y_2. The subscripts indicate whether the component value is an increment (i or $j = 1$), or decrement ($i, j = -1$) relative to a standard value ($i, j = 0$). Only one of the components will contain a change from the standard value in a single stimulus. If the stimulus presented is either S_{10} (an increment on component Y_1 relative to the standard value with no change from the standard on component Y_2) or S_{01} (an increment on component Y_2 relative to the standard value), the correct response is to indicate that an "increment" has occurred, otherwise respond "decrement". Signal detection theory, and its multidimensional counterpart, *general recognition theory* (GRT), assumes that when a stimulus is presented, it generates an internal percept that is modeled as a sample from a probability density defined on the perceptual analog(s) of the stimulus component(s) (e.g., Ashby, 1992; Ashby & Townsend, 1986; Green & Swets, 1966; Wickens, 1992).

We assume that these perceptual densities are bivariate Gaussian (normal). Each stimulus distribution can be completely specified by its mean vector $\boldsymbol{\mu}_{ij}$ and variance–covariance matrix $\boldsymbol{\Sigma}_{ij} = \begin{pmatrix} \sigma^2_{y_1} & \mathrm{Cov}\,(Y_1, Y_2) \\ \mathrm{Cov}\,(Y_1, Y_2) & \sigma^2_{y_2} \end{pmatrix}_{ij}$. Given this setup, the task is equivalent to a classification task which assigns percepts (y_1, y_2) to category responses $R_{\text{increment}}$ or $R_{\text{decrement}}$. We articulate different decision strategies which map percepts to category responses from which predicted response probabilities can be computed. These response probabilities are then used to generate predictions for sensitivity in both *certainty* and *uncertainty* conditions and RMS to compare their relative performance.

In typical applications of this task, a sensitivity measure, $d'_{\text{empirical}}$, for each component is computed from observed hits and false alarms in both the *certainty* and *uncertainty* conditions in the following manner. Consider the sensitivity on the Y_1 component. A hit occurs when the observer indicates "increment" ($R_{\text{increment}}$) when S_{10} has been presented. The hit rate is the proportion of increment responses given the stimulus S_{10}, $P(R_{\text{increment}}|S_{10})$. A false alarm occurs when the observer indicates "increment" when S_{-10} has been presented. Thus, the false alarm rate is $P(R_{\text{increment}}|S_{-10})$. The empirically computed sensitivity along component Y_1, is found from

$$d'_{\text{empirical},\,Y_1} = \Phi^{-1}\left[\mathrm{P}\left(R_{\text{increment}}|S_{10}\right)\right] - \Phi^{-1}\left[\mathrm{P}\left(R_{\text{increment}}|S_{-10}\right)\right] \tag{1}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse cumulative distribution function of the standard univariate Gaussian (normal) random variable (e.g., MacMillan & Creelman, 2005). The sensitivity for component Y_2 is computed analogously from the relevant hits and false alarms. The main purpose of this paper it to derive predictions for component sensitivity measures in both the certainty and uncertainty tasks that result from different decision model and perceptual representation combinations. We use these model representations to compute the response probabilities input into Eq. (1).

We assume the following holds for the perceptual distributions in the derivations that follow. We assume a form of *stimulus*