Journal of Mathematical Psychology 66 (2015) 83–102

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Conjunction and negation of natural concepts: A quantum-theoretic modeling

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HIGHLIGHTS

- We study the effects of negation on conceptual conjunction and perform two experiments to test such effects.
- We analyze the collected data and show that they cannot be modeled in a classical probability framework.
- We develop a quantum-theoretic approach to model conjunction and negation of two concepts.
- We show that a quantum probability model correctly describes the collected data.
- We explain the results above in terms of genuine quantum effects, i.e. interference and conceptual emergence.

ARTICLE INFO

Article history:
Received 6 June 2014
Received in revised form
27 January 2015
Available online 12 March 2015

Keywords:
Concept combinations
Vagueness
Negated concept
Overextension
Quantum cognition
Quantum modeling

ABSTRACT

We perform two experiments with the aim to investigate the effects of negation on the combination of natural concepts. In the first experiment, we test the membership weights of a list of exemplars with respect to two concepts, e.g., *Fruits* and *Vegetables*, and their conjunction *Fruits And Vegetables*. In the second experiment, we test the membership weights of the same list of exemplars with respect to the same two concepts, but negating the second, e.g., *Fruits* and *Not Vegetables*, and again their conjunction *Fruits And Not Vegetables*. The collected data confirm existing results on conceptual combination, namely, they show dramatic deviations from the predictions of classical (fuzzy set) logic and probability theory. More precisely, they exhibit conceptual vagueness, gradeness of membership, overextension and double overextension of membership weights with respect to the given conjunctions. Then, we show that the quantum probability model in Fock space recently elaborated to model Hampton's data on concept conjunction (Hampton, 1988a) and disjunction (Hampton, 1988b) faithfully accords with the collected data. Our quantum-theoretic modeling enables to describe these non-classical effects in terms of genuine quantum aspects, namely 'contextuality', 'superposition', 'interference' and 'emergence'. The obtained results confirm and strengthen the analysis in Aerts (2009a) and Sozzo (2014) on the identification of quantum structures in experiments on conceptual vagueness. And, more, they can be inserted within the general research on the identification of quantum structures in cognitive processes.

1. Introduction

In the last years there has been a renewed interest in the formulation of a unified psychological theory for representing and structuring concepts. Indeed, traditional approaches to concept theory, mainly, 'prototype theory' (Rosch, 1973, 1978, 1983), 'exemplar theory' (Nosofsky, 1988, 1992) and 'theory theory' (Murphy & Medin, 1985; Rumelhart & Norman, 1988) are still facing a crucial difficulty, namely, 'the problem of how modeling the combination of two or more natural concepts starting from the modeling of the component ones'. This 'combination problem' has been revealed by several cognitive experiments in the last thirty years. More precisely:

(i) The 'Guppy effect' in concept conjunction, also known as the 'Pet-Fish problem' (Osherson & Smith, 1981). If one measures the typicality of specific exemplars with respect to the concepts *Pet* and *Fish* and their conjunction *Pet-Fish*, then one experimentally finds that an exemplar such as *Guppy* is a very typical example of *Pet-Fish*, while it is neither a very typical example of *Pet* nor of *Fish*.

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http://dx.doi.org/10.1016/j.jmp.2015.01.005
0022-2496/

(ii) The deviation from classical (fuzzy) set-theoretic membership weights of exemplars with respect to pairs of concepts and their conjunction or disjunction (Hampton, 1988a,b). If one measures the membership weight of an exemplar with respect to a pair of concepts and their conjunction (disjunction), then one experimentally finds that the membership weight of the exemplar with respect to the conjunction (disjunction) is greater (less) than the membership weight of the exemplar with respect to at least one of the component concepts.

(iii) The so-called 'borderline contradictions' (Alxatib & Pelletier, 2011). Roughly speaking, a borderline contradiction is a sentence of the form $P(x) \wedge \neg P(x)$, for a vague predicate P and a borderline case x, e.g., the sentence "Mark is rich and Mark is not rich".

If one accepts that concepts are 'graded', or 'fuzzy', notions (Osherson & Smith, 1982; Zadeh, 1965, 1982), as empirical evidence seem to confirm, then one cannot represent the membership weights and typicalities expressing such gradeness in a classical (fuzzy) set-theoretic model, where conceptual conjunctions are represented logical conjunctions and conceptual disjunctions are represented by logical disjunctions. These difficulties affect both 'extensional' membership-based (Rips, 1995; Zadeh, 1982) and 'intensional' attribute-based (Hampton, 1988b, 1997; Minsky, 1975). This combination problem is considered so serious that many authors maintain that not much progress is possible in the field if no light is shed on this problem (Fodor, 1994; Hampton, 1997; Kamp & Partee, 1995; Komatsu, 1992; Rips, 1995). However no mechanism and/or procedure has as yet been identified that gives rise to a satisfactory description or explanation of the effects appearing when concepts combine.

Very similar effects and deviations from the predictions of traditional approaches have meanwhile been experienced in other domains of cognitive science, specifically, in behavioral economics and decision theory. These and other difficulties have led various scholars to look for alternative approaches which could provide a more satisfactory picture of 'what occurs in human thought in a cognitive process'. Among the possible alternatives, a major candidate is what has been called 'quantum cognition' and it rests the application of the mathematical formalism of quantum theory in cognitive and social domains (see. e.g. Aerts, 2009a,b; Aerts, Broekaert, Gabora, & Sozzo, 2013; Aerts & Czachor, 2004; Aerts & Gabora, 2005a,b; Aerts, Gabora, & Sozzo, 2013; Aerts & Sozzo, 2011, 2014; Aerts, Sozzo, & Tapia, 2014; Busemeyer & Bruza, 2012; Busemeyer, Pothos, Franco, & Trueblood, 2011; Haven & Khrennikov, 2013; Khrennikov, 2010; Pothos & Busemeyer, 2009, 2013; Van Rijsbergen, 2004; Wang, Busemeyer, Atmanspacher, & Pothos, 2013).

In this paper, we mainly deal with the quantum-theoretic approach to cognitive science elaborated in Brussels. This approach was motivated by a two decade research on the foundations of quantum theory (Aerts, 1999), the origins of quantum probability (Aerts, 1986; Pitowsky, 1989) and the identification of typically quantum aspects in the macroscopic world (Aerts & Aerts, 1995; Aerts, Aerts, Broekaert, & Gabora, 2000). A *State Context Property*, or *SCoP*, formalism was worked out within the Brussels approach which relies on the interpretation of a concept as an 'entity in a specific state changing under the influence of a context' rather than as a 'container of instantiations' (Aerts & Gabora, 2005a,b), and allowed the authors to provide a quantum representation of the guppy effect (Aerts & Gabora, 2005a,b). Successively, the mathematical formalism of quantum theory was employed to model the overextension and underextension of membership weights measured by Hampton (1988a,b). More specifically, the overextension for conjunctions of concepts measured by Hampton (1988a) was described as an effect of quantum interference, superposition and emergence (Aerts, 2009a; Aerts, Gabora, & Sozzo, 2013), which also play a primary role in the description of both overextension and underextension for disjunctions of concepts (Hampton, 1988b). Furthermore, a specific conceptual combination experimentally revealed the presence of another genuine quantum effect, namely, entanglement (Aerts, 2009a,b; Aerts, Broekaert et al., 2013; Aerts, Gabora, & Sozzo, 2013; Aerts & Sozzo, 2011). Finally, this quantum-theoretic framework was successfully applied to describe borderline vagueness (Sozzo, 2014).

More specifically, in the present paper we generalize the analysis in Aerts (2009a) of Hampton's overextension for the conjunction of two concepts, extending it to conjunctions and negations. Negative concepts have been typically considered as 'singular concepts', since they do not have a prototype. Indeed, it is, for example, easy to determine the membership of a concept such as *Not Fruit*, but it does not seem that such a determination involves similarity with some prototype of *Not Fruit*. This is why one is naturally led to derive the negation of a concept from (fuzzy set) logical operations on the positively defined concept. There has been very little research on how human beings interpret and combine negated concepts. In this respect, Hampton (1997) performed a set of experiments in which he considered both conjunctions of the form *Tools Which Are Also Weapons* and conjunctions of the form *Tools Which Are Not Weapons*. As expected, his seminal work confirmed overextension in both conjunctions, also showing a violation of Boolean classical logical rules for the negation. These results were the starting point for our research in this paper, whose content can be summarized as follows.

In Section 2 we describe and statistically analyze the two experiments we performed. In the first experiment, we tested the membership weights of four different sets of exemplars with respect to four pairs (A, B) of concepts and their conjunction 'A and B'. In the second experiment, we tested the membership weights of the same four sets of exemplars with respect to the same four pairs (A, B) of concepts, but negating the second concept, hence actually considering A, 'not B' and the conjunction 'A and not B'. We observe that, already at this level, several exemplars exhibited overextension with respect to both 'A and B' and 'A and not B', hence we get a first clue that a deviation from classical (fuzzy set) logic and probability theory is at play in our experiments. A complete analysis of the 'non-classicality' underlying the collected data is presented in Section 2.4 where we prove two theorems on the representability of a given set of experimental data in a classical Kolmogorovian probability space, thus extending the analysis in Aerts (2009a) to negated concepts. By applying these theorems, we show that a large part of our data cannot be modeled in a classical Kolmogorovian space. Moreover, we notice that the deviations from classicality are of two types: (i) overextension of membership weights with respect to both conjunctions 'A and B' and 'A and not B', (ii) deviation of the negation 'not B' of the concept B from the classical logical negation. This non-classical behavior led us to inquire into the possibility of representing our data in a quantum-mechanical framework. After a brief overview of the rules of a quantum-theoretic modeling in Section 3, we develop this modeling for the combinations 'A and B' and 'A and not B' in Section 4, thus extending the analysis in Aerts (2009a). We draw our conclusions in Section 5, where we:

(i) prove that a large number of the collected data can be represented in our quantum-theoretic modeling in Fock space;

(ii) describe the observed deviations from classicality as a consequence of genuine quantum effects, such as, 'contextuality', 'interference', 'superposition' and 'emergence';

(iii) provide a further support to the explanatory hypothesis we have recently put forward for the effectiveness of a quantum approach in cognitive and decision processes. According to