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Possibility to agree on disagree from quantum information and decision making

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HIGHLIGHTS

- Quantum model for representation of states of the world, knowledge, and common knowledge was elaborated.
- Quantum decision making: the possibility to agree on disagree — even with common prior.
- Classical Aumann theorem can be recovered under mathematically nontrivial conditions of compatibility.

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ABSTRACT

The celebrated Aumann theorem states that if two agents have common priors, and their posteriors for a given event E are common knowledge, then their posteriors must be equal; agents with the same priors cannot agree to disagree. The aim of this note is to show that in some contexts agents using a quantum probability scheme for decision making can agree to disagree even if they have the common priors, and their posteriors for a given event E are common knowledge. We also point to sufficient conditions guaranteeing impossibility to agree on disagree even for agents using quantum(-like) rules in the process of decision making. A quantum(-like) analog of the knowledge operator is introduced; its basic properties can be formulated similarly to the properties of the classical knowledge operator defined in the set-theoretical approach to representation of the states of the world and events (Boolean logics). However, this analogy is just formal, since quantum and classical knowledge operators are endowed with very different assignments of truth values. A quantum(-like) model of common knowledge naturally generalizing the classical set-theoretic model is presented. We illustrate our approach by a few examples; in particular, on attempting to escape the agreement on disagree for two agents performing two different political opinion polls. We restrict our modeling to the case of information representation of an agent given by a single quantum question-observable (of the projection type). A scheme of extending of our model of knowledge/common knowledge to the case of information representation of an agent based on a few question-observables is also presented and possible pitfalls are discussed.

1. Introduction

Aumann's approach Aumann (1976) to common knowledge and his "no agreement on disagree theorem" played an important role in creation of a proper mathematical model of common knowledge, see also Brandenburger and Dekel (1987), Geanakoplos (1994), Hild, Jeffrey, and Risse (1987), McKelvey and Page (1986), Monderer and Samet (1989) and Nielsen (1984), for generalizations.

The main puzzle raised by Aumann's theorem and its generalizations is that people often "agree on disagree"; so the natural question arises: *How to explain differences in beliefs*? (see, e.g., Aumann, 1976, Vanderschraaf & Sillari, 2013 for discussion). The simplest solution is to deny the possibility that decision makers are able to set common priors. However, in many situations the common prior assumption is very natural, since assignment of common priors is based on sharing common information. As was pointed in Vanderschraaf and Sillari (2013), "another way Aumann's result might fail is if agents do not have common knowledge that they update their beliefs by Bayesian conditionalization. Then clearly, agents can explain divergent opinions as the result of others having modified their beliefs in the "wrong" way".

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Of course, the latter explanation is based on consideration of Bayesian updating as the "right" way updating. The reduction of rationality to "Bayesian rationality" is an important assumption of classical decision theory. As we shall discuss later in more detail, this assumption is equivalent to the assumption that human beings process information by using the rules of *Boolean logic*. In this note we show that agents using more general logic, so called quantum logic, in information processing (see Khrennikov, 2004b for discussion), can "agree on disagree". And they would not update their beliefs in the "wrong" way, since they all apply (at least heuristically) another common rule for probability update, based on the laws of quantum information and probability.

We remark that during recent years the mathematical formalism of quantum mechanics was widely applied to problems of decision making, see, e.g., Aerts, Gabora, and Sozzo (in press), Aerts, Sozzo, and Tapia (2012), Asano, Basieva, Khrennikov, and Ohya (2012), Asano, Basieva, Khrennikov, Ohya, and Tanaka (2012), Asano, Basieva, Khrennikov, Ohya, and Yamato (2013), Asano, Ohya, Tanaka, Khrennikov, and Basieva (2011a,b), Brandenburger (2010); Busemeyer and Bruza (2012), Busemeyer, Pothos, Franco, and Trueblood (2011); Busemeyer, Santuy, and Lambert-Mogiliansky (2008), Busemeyer and Townsend (1993), Busemeyer, Wang, and Lambert-Mogiliansky (2009); Busemeyer, Wang, and Townsend (2006), Cheon and Takahashi (2010), Dzhafarov and Kujala (2012a,b), Haven and Khrennikov (2009, 2012), Khrennikov (2009, 2010, 2011), Khrennikova (2012, 2013), Lambert-Mogiliansky, Zamir, and Zwirn (2009), McKelvey and Page (1986), Monderer and Samet (1989), Moore (2002), Nielsen (1984), Penrose (2002), Pothos and Busemeyer (2009, 2013), Pothos, Busemeyer, and Trueblood (2013), Trueblood and Busemeyer (2011) and Wang and Busemeyer (2013).[1] This project is a part of a more general project on quantum(-like) modeling of cognition (de Barros & Suppes, 2009; Accardi & Boukas, 2006; Aerts et al., in press, 2012; Asano, Basieva, Khrennikov, & Ohya, 2012; Asano, Basieva, Khrennikov, Ohya, et al., 2012; Asano et al., 2012a,b, 2013; Asano, Basieva, Khrennikov, Ohya, & Yamato, 2013; Asano et al., 2011a,b; Atmanspacher & Filk, 2012; Atmanspacher & Römer, 2012; Aumann, 1976, 1995; Basieva et al., 2011; Binmore & Brandenburger, 1988; Bruza & Cole, 2005; Busemeyer & Bruza, 2012; Busemeyer et al., 2011, 2008; Busemeyer & Townsend, 1993; Busemeyer et al., 2009, 2006; Cheon & Takahashi, 2010; Conte et al., 2009, 2007; Dzhafarov & Kujala, 2012a,b, 2013, 2014, 0000; Haven & Khrennikov, 2009, 2012; Khrennikova, 2013; Lambert-Mogiliansky et al., 2009; McKelvey & Page, 1986; Monderer & Samet, 1989; Moore, 2002; Nielsen, 1984; Penrose, 2002; Pothos & Busemeyer, 2009, 2013; Pothos et al., 2013; Trueblood & Busemeyer, 2011). The latter is based on the *quantum-like paradigm* (Khrennikov, 2010) that information processing by complex cognitive systems (including social systems) taking into account contextual dependence of information and probabilistic reasoning can be mathematically described by quantum information and probability theories, see Appendix A.2 for a discussion "whether quantum features are in the outside world or in the mind of people (quantum cognition)".

We remark that from the logical viewpoint, usage of quantum formalism implies violation of laws of classical (Boolean) logics. This viewpoint was presented already in the pioneer monograph of Von Neuman (1955), see Birkhoff and von Neumann (1936) for the detailed presentation. Thus from this viewpoint cognitive systems can violate the laws of Boolean logic and follow the laws of more general "quantum logic".

How can one find evidences of violations of classical logic? Since classical probability theory (Kolmogorov's measure-theoretic axiomatics, 1933) is based on Boolean logic, then possible departures from classical logic can be seen in violations of the basic laws of classical probability theory, see Khrennikov (2004a) for an extended discussion. One of such laws is the *law of total probability*. Its violation have been found in various sets of statistical data, e.g., for recognition of ambiguous figures, Cheon and Takahashi (2010), Conte et al. (2009, 2007), Khrennikov (2010), for the disjunction effect (related to violation of the Savage sure principle and, hence, playing an important role in economics, see, e.g., Hofstader (1983), Tversky and Shafir (1992), Shafir and Tversky (1992), Croson (1999), Kahneman (2003)), Aerts et al. (2012), Asano, Basieva, Khrennikov, and Ohya (2012), Asano, Basieva, Khrennikov, Ohya, et al. (2012), Asano, Basieva, Khrennikov, Ohya, and Yamato (2013), Asano et al. (2011a,b), Busemeyer and Bruza (2012), Busemeyer and Townsend (1993), Busemeyer et al. (2011, 2008, 2009, 2006), Haven and Khrennikov (2009, 2012), Khrennikov (2009, 2010), Pothos and Busemeyer (2013) and Pothos et al. (2013), see also Khrennikov (2004a), Khrennikova (2013) for other theoretical and experimental studies of violations of the law of total probability outside of physics. We remark that violations of law of total probability in quantum physics were discussed by many authors, in particular, in Feynman and Hibbs (1965), see also Khrennikov (2003) and references herein.

In this paper we show that the quantum generalization of the Bayesian updating leads to violation of the celebrating *Aumann theorem* which states that *if two agents have the common priors, and their posteriors for a given event E are common knowledge, then their posteriors must be equal; agents with the same priors cannot agree to disagree*. We show that in *some contexts* agents using quantum logic can *agree to disagree* even if they have the common priors, and their posteriors for a given event E are common knowledge.

One of the departures from the classical Aumann's model is the existence of *incompatible information representations of the world* by different agents. Instead of the set-theoretical (Boolean) partitions of the space of the states of the world Ω, we consider partitions of the unit operator in complex Hilbert space H (space of the quantum states of the world) consisting of the mutually orthogonal projectors. In general these partitions can be incompatible, i.e., the corresponding question-operators of different agents need not commute.

We point out that incompatibility of information representations of different agents is not the only quantum feature of the model generating the possibility to agree on disagree. We show by an example having nontrivial cognitive and psychological (as well as sociological) content, Section 7, that the Aumann's theorem can be violated even for commuting question-operators of agents. This example was motivated by Moore (2002) political pool studies on honesty of Bill Clinton and Al Gore. Moore studied the order effect. As was shown in Wang and Busemeyer (2013), the corresponding statistical data exhibits nonclassical features and can be represented with the aid of incompatible observables (see also Busemeyer & Bruza, 2012 on a general discussion on quantum representation of order effects in psychology). We would like to make compatible these question-observables: $A^{(1)}$ = "Is Bill Clinton honest and trustworthy?" and $A^{(2)}$ = "Is Al Gore honest and trustworthy?". To do this, we associate them with two different agents who perform two different political polls. The first one is based (solely) on the question $A^{(1)}$ and the second one on the question $A^{(2)}$. Here the order effect disappear and the question-observables $A^{(i)},\ i = 1, 2$, can be represented by commuting operators, $[A^{(1)},\ A^{(2)}] = 0$. And in Section 7 we demonstrate that even

[1] We remark that the framework of quantum(-like) decision making has interesting applications not only in psychology, cognitive science, and social science, but even in molecular biology, where a cell is considered as a kind of decision maker, see Asano et al. (2012a,b), Asano, Basieva, Khrennikov, Ohya, and Yamato (2013), Asano et al. (2013), Atmanspacher and Filk (2012), Atmanspacher and Römer (2012), Aumann (1976, 1995), Basieva, Khrennikov, Ohya, and Yamato (2011) and Binmore and Brandenburger (1988). In principle, such an approach can be interpreted as the first step towards mathematical modeling of cell's cognition.