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Means and standard deviations, or locations and scales? That is the question!

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ABSTRACT

According to standard experimental practice, researchers randomly assign participants to experimental and control conditions, deeming the experiment "successful" if the means of the two conditions differ in the hypothesized direction. Even for complex experiments, with many conditions, success generally depends on a comparison or contrast of means across conditions. Because the experimental manipulation may change the shape of the distribution, we show that a difference in means, even if large and in the hypothesized direction, does not necessarily indicate the success of the experiment. To make this determination, it also is necessary to compute location statistics. It is possible for means to change but for locations not to change, for means not to change but for locations to change, and for mean differences and location differences to be in opposite directions. Therefore, typical research that depends on differences between means across conditions, cannot be trusted in the absence of location statistics. For similar reasons, nor can standard deviations be trusted without scale statistics. Therefore, we take the radical step of arguing that all researchers who report means and standard deviations, also should be required to report corresponding location and scale statistics.

1. Introduction

The standard experimental approach is similar across most sciences. At minimum, the researcher performs an experimental manipulation that includes an experimental condition and a control condition, and a difference in means, in the hypothesized direction, across the two conditions, is taken as indicating "success." The researcher also might compute an effect size. Usually, this implies dividing the difference between means by the standard deviation, to find the size of the difference in standard deviation units. Of course, much research is more sophisticated, with more conditions, but the basic procedure of comparing means across conditions, and drawing conclusions about the success or failure of the experiment from differences between means, remains. Our goal is to show that using means in this way is problematic, but also that there is a solution.

To introduce the problem, it is important to realize that most distributions are skewed rather than normal (Ho & Yu, 2015; Micceri, 1989). In addition, in the context of an experiment, it is quite possible that the researcher's experimental manipulation changes the skew of the distribution in the experimental condition relative to the control condition. The experimental manipulation might introduce ceiling effects, floor effects, extreme scores, or other factors that increase the level of skew relative to the control condition. Alternatively, the control condition might have a skewed distribution and the experimental manipulation might decrease, eliminate, or reverse the skew in the experimental condition. The possibility that an experimental manipulation can cause the experimental and control conditions to differ with respect to skewness suggests that there is an alternative explanation for differences in means across these conditions, other than that the experiment was successful. This alternative explanation constitutes the present main topic.

To gain an initial understanding of the argument to be developed more formally later, consider the familiar mean, median, and mode. These are different ways of assessing what might be considered, informally, to be the "center" or "location" of a distribution. That there are multiple such assessments suggests that the center or location of a distribution, used in this general sense, is not well defined, though we will use location in a very specific, and well-defined sense, later. Intuitively, the larger the center or location of a distribution; such as the mean, median, or mode; the more the distribution is shifted to the right on a horizontal axis. But there is an important caveat in the context of skew-normal distributions.

Skew-normal distributions are defined by three parameters, to be elaborated later: location (the "center" of the distribution), scale (dispersion of scores), and skewness (shape of the distribution). Because location is one of the three defining parameters of skew-normal distributions; and mean, median, and mode are not; our focus is on location. It is vital not to confuse the word, "location," as used generally to indicate a class of parameters, such as mean, median, and mode; versus used specifically, as a parameter that helps define a skew-normal

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distribution. Hereafter, we use "location" in the latter sense.

Just as "location," in the context of skew-normal distributions, differs from mean; "scale" differs from standard deviation. Because scale is a defining characteristic of skew-normal distributions, and standard deviation is not, scale is more useful than standard deviation in understanding skew-normal distributions.

Location, scale, and skewness are independent parameters of skew-normal distributions. Consequently, it is quite possible for an experimental manipulation to influence one of them, without influencing the others; or to influence two of them, without influencing the third. For example, an experimental manipulation could influence the skewness parameter, which necessarily influences the mean and standard deviation, if the location and scale parameters are unchanged. In that case, with the location of the skew-normal distribution unchanged, it follows that what seems a successful experimental manipulation based on means, is not successful after all, based on locations.

The present argument relates to one made by Speelman and McGann (2013), who showed that any summary statistic might be misleading. In fact, although our present goal is to support the use of location and scale statistics to uncover when means and standard deviations are misleading, we also wish to be up front that even location or scale statistics can be misleading. For example, if one has a bimodal distribution rather than a skew-normal distribution, the location is misleading. This is one reason we encourage researchers to use visual displays to aid in better understanding their data (Valentine, Aloe, & Lau, 2015).

2. The alternative explanation

It is necessary to provide a brief introduction to the family of skew-normal distributions, of which the family of normal distributions is a subset (Azzalini, 2014). A random variable Z is said to have a standard skew-normal distribution with *skewness* parameter λ, denoted as $Z \sim SN(\lambda)$, if its probability density function (*pdf*) is given by

$$f(z) = 2\phi(z)\Phi(\lambda z), \tag{1}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function (*pdf*) and cumulative distribution function (*cdf*) of the standard normal distribution, respectively.

Let $Z \sim SN(\lambda)$, and consider the linear function of Z

$$X = \xi + \omega Z. \tag{2}$$

Then the random variable X has a skew-normal distribution with *location* parameter ξ, *scale* parameter ω, and *skewness* parameter λ, denoted as $X \sim SN(\xi, \omega^2, \lambda)$. The *pdf* of X is given by

$$f(x) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\lambda\frac{x-\xi}{\omega}\right). \tag{3}$$

And the mean and variance are:

$$\mathrm{E}(X) = \mu = \xi + \sqrt{\frac{2}{\pi}}\delta\omega \quad \text{and} \quad \mathrm{V}(X) = \sigma^2 = \omega^2\left(1 - \frac{2}{\pi}\delta^2\right), \tag{4}$$

where $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$.

The foregoing equations indicate important implications. When the skewness parameter $\lambda = 0$, the distribution is normal. In turn, the mean parameter μ and the location parameter ξ are equivalent. Thus, the mean parameter also functions as the location parameter. However, when $\lambda \neq 0$, $\mu \neq \xi$; the distribution is skew-normal and the mean fails to give the location of the distribution. In addition, when the distribution is normal, the standard deviation parameter σ also functions as the scale parameter ω, but when the distribution is skew-normal, $\sigma \neq \omega$, and the standard deviation σ fails to function as the scale ω. The present argument depends on the fact that for skewed distributions ($\lambda \neq 0$), $\mu \neq \xi$ and $\sigma \neq \omega$.

Returning to the case where a researcher performs an experiment, suppose that the effect of the experimental manipulation is to increase the skewness (in the positive direction) of the experimental condition relative to the control condition. But let us also assume that the location ξ is the same in both conditions. If the researcher wishes to increase the distribution of scores in the experimental condition relative to the distribution of scores in the control condition, it should be obvious that the experiment is not successful because the location is the same in both conditions. Nevertheless, the means are necessarily different, and in the hypothesized direction too. Stated more generally, changing the skewness of a distribution causes the mean to change too, *if the location and scale remain the same*. And we arrive at our alternative explanation for a larger mean in the experimental condition than in the control condition. That is, it could be that the experiment is not successful because the location does not change, but the experiment *seems* successful because the mean changes in the predicted direction. Furthermore, the change in skewness also forces a change in the standard deviation, if the scale does not change. In general, increasing the skewness magnitude in either the positive or negative direction forces the standard deviation to decrease when the scale remains constant. Fig. 1 illustrates this alternative explanation. In Fig. 1, all the curves have the same location and scale, but also have different shapes (amounts of skew), and consequently, different means and standard deviations. More precise mathematical demonstrations are forthcoming.

It is possible to increase the precision of the argument by introducing Figs. 2–4. To create these figures, we mathematically modeled an experiment with experimental and control conditions, stipulating that the control condition distribution is normal ($\lambda = 0$), with location (and

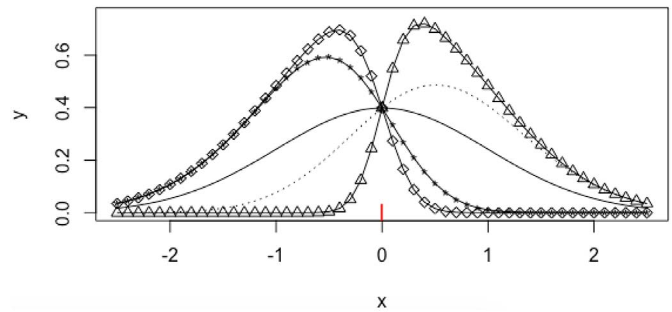

Fig. 1. The probability density functions (*pdf*) for skew-normal distributions are shown with location parameter 0 (pointed by '|'); scale parameter 1; and skewness parameters −4 (cube curve), −2 (star curve), 0 (solid curve), 1 (dot curve) and 5 (triangle curve).

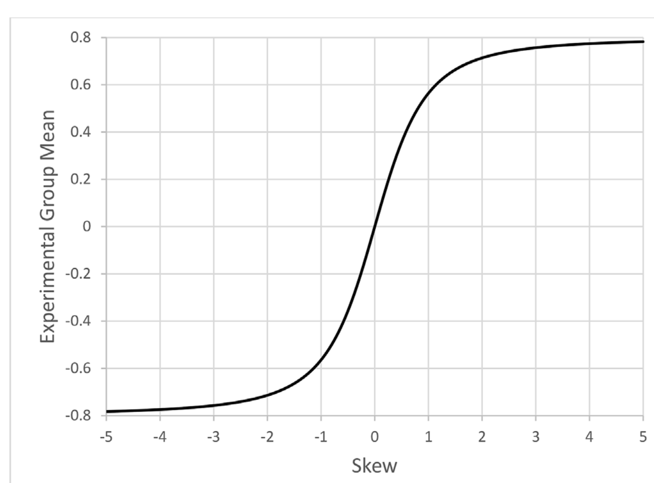

Fig. 2. The mean of the experimental condition is presented along the vertical axis as a function of its skew, presented along the horizontal axis. The location is 0 and the scale is 1.