



Integrating intuitive and novel grounded concepts in a dynamic geometry learning environment



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ABSTRACT

The development of geometry knowledge requires integration of intuitive and novel concepts. While instruction may take many representational forms we argue that grounding novel information in perception and action systems in the context of challenging activities will promote deeper learning. To facilitate learning we introduce a *grounded integration pattern* of instruction, focusing on (1) eliciting intuitive concepts, (2) introducing novel grounding metaphors, and (3) embedding challenges to promote distinguishing between ideas. To investigate this pattern we compared elementary school children in two conditions who engaged in variations of a computer-based dynamic geometry learning environment that was intended to elicit intuitive concepts of shapes. In the *grounded integration* condition children performed a procedure of explicitly identifying defining features of shapes (e.g. right angles) with the assistance of animated depictions of spatially-meaningful gestures (e.g. hands forming right angles). In a *numerical integration* condition children identified defining features with the assistance of a numerical representation. Children in the *grounded integration* were more likely to accurately identify target shapes in a posttest identification task. We discuss the relevancy of the *grounded integration pattern* on the development of instructional tools.

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1. Introduction

Geometry is a critical, yet often overlooked, branch of mathematics education. Although the [National Council Of Teachers Of Mathematics \(2000\)](#) stresses the value of geometry instruction, particularly poor U.S. performance on geometry items of standardized assessments, relative to other developed nations ([Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005](#)), likely reflects inadequate treatment of geometry in traditional curricula ([Clements, 2004](#)).

Overwhelmingly, traditional instructional activities rely on static images and simplistic tasks. Consequently, students are under-prepared for higher-level geometric reasoning ([Clements, 2004](#); [Clements & Battista, 1992](#)). As a potential remedy, interactive geometry software, such as Logo, enables students to apply a constructive, inquiry-based approach to mathematics ([Papert, 1980](#); [Papert, Watt, DiSessa, & Weir, 1979](#)). Research spanning from Piaget ([Piaget & Inhelder, 1956](#); [Piaget, Inhelder, & Szeminska, 1960](#)) to more recent theory regarding “embodied” cognition ([Lakoff & Núñez, 2000](#), pp. xvii, 493) suggests that young children’s early knowledge of geometry is primarily visuo-spatial in nature. Therefore, the concrete representations of mathematical concepts embedded in interactive geometry software should afford a greater intuitive foothold than traditional materials.

Yet, in spite of this clear theoretical support, research on the use of concrete representations in education is inconclusive. For example, while Logo generated much early enthusiasm, a number of studies show mixed or negative findings on learning outcomes ([Howe, O’Shea, & Plane, 1979](#); [Hughes & Greenbough, 1995](#); [Johnson, 1986](#); [Noss & Hoyles, 1992](#); [Simmons & Cope, 1990](#)). In contrast, a number of recent studies highlight benefits for educational materials that are more abstract in design ([Kaminski, Sloutsky, & Heckler, 2009](#); [McNeil & Uttal, 2009](#)).

Given the theoretical support for concrete representations, what accounts for their unevenness in practice? One possibility is that concrete representations are, in fact, too intuitive. Specifically, the additional affordances of concrete representations facilitate problem

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solving without necessarily attending to or reflecting upon deeper structure of the materials. Symbolic approaches avoid unintended consequences by introducing only those features that are relevant to learning goals, thereby facilitating transfer to structurally-similar problems (Kaminski, Sloutsky, & Heckler, 2008). In more extreme cases intuitive concepts may even introduce misconceptions that are then applied to novel materials (Ohlsson, 2009).

While in some cases abstract materials may afford a “symbolic advantage” (Koedinger, Alibali, & Nathan, 2008), in other cases the lack of familiar or intuitive representations can open a rift between the mathematics of school and everyday experience (Lave, 1988). In the approach taken here, rather than shifting instruction towards more symbolic forms to avoid intuitive concepts, we focus on introducing new visuo-spatial, or grounded, representations that can be compared with intuitive concepts, directly. In the following we describe a *grounded integration pattern* of instruction which aims to promote coordination between intuitive and novel, spatially-grounded representations of geometry. We detail how this approach is supported by recent cognitive-developmental research and how current digital technologies support its application.

1.1. The emerging cognitive science of geometry

Fundamental processes that support mathematical thinking emerge consistently early across individuals and cultures. In the numerical domain core knowledge emerges from universal abilities to judge magnitude and recognize numerosity of small collections (Feigenson, Dehaene, & Spelke, 2004), which is supported by distinct system of neural structures (Dehaene, 1997). Beyond the core knowledge that grounds intuitive concepts, environmental factors, such as SES, can promote or inhibit further development of formal mathematical ideas (Booth & Siegler, 2006). Promisingly, deficits can be addressed through motivating activities, such as board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009). Clearly, instruction plays a fundamental role in supporting attainment of higher-level ideas.

Likewise, in the domain of geometry researchers are beginning to address development in terms of core knowledge. Spelke, Lee, and Izard (2010) claim that geometry concepts are grounded in two core systems associated with spatial navigation and object perception. From this perspective the concept of a shape, e.g. a *rectangle*, is derived from the experience of viewing rectangular objects, such as the face of a block, or navigating along a rectangular path. In particular, object perception fosters concepts of distance and angle, while navigation fosters concepts of distance and directional sense (i.e., left vs. right).

Consistent with Spelke et al.'s. (2010) framework, years of perceptual research suggests that individuals develop geometric ideas by exposure to common object and images. For example, Bomba and Siqueland (1983) found that 3-month-olds recognized a prototypical triangle after viewing a series of triangles, which varied randomly from the prototype, without actually seeing the prototype. In turn, internalized prototypical representations may then be applied to identify and classify novel figures.

Although prototypes facilitate development of geometric ideas at an early age, prototypical representations can hinder later development of formal concepts. In particular, individuals tend to classify images based on perceptually-salient cues, such as symmetry (Quinlan & Humphreys, 1993), dispersion (irregularity), elongation, and jaggedness (Behrman & Brown, 1968). While geometric prototypes inherently contain defining features, they also exhibit these salient secondary features (e.g. elongation of a prototypical rectangle) which may drive conceptualization.

In school-based tasks young children often apply informal labels, such as “slanty”, “pointy”, or “skinny” to describe common shapes (Clements, Swaminathan, Hannibal, & Sarama, 1999). While there is value in facilitating description of geometric figures with informal language, children are unlikely to recognize differences between secondary and defining features. In this case the most salient perceptual feature of a figure strongly influences its classification at the expense of any formal criteria. For example, in identification tasks many children mistake elongated (non-rectangular) parallelograms for rectangles, while missing squares (Burger & Shaughnessy, 1986; Clements et al., 1999).

Although irrelevant salient perceptual characteristics often influence children's thinking, there is evidence that more normative concepts are within reach. Specifically, Dehaene and Izard (2006) found that members of an Amazonian tribe, with little or no exposure to formal concepts in geometry, could distinguish between parallel and non-parallel, as well as perpendicular and non-perpendicular line segments – important concepts in Euclidean geometry. While students may have the perceptual ability to make these distinctions, instruction is necessary to promote their relevance to shape concepts.

1.2. Grounded instruction in geometry

The tendency for younger children to identify objects based on superficial characteristics is not unique to geometry. Children often classify common objects – such as taxicabs and islands – based on non-distinguishing, but salient characteristics (Keil & Baterman, 1984). On the other hand, adults or domain experts tend to classify objects based on abstract or hidden structure (Carey, 1985; Chi, Feltovich, & Glaser, 1981) and perceptually-abstract analytical systems (Mandler, 1992; Sloman, 1996). This is often described as a general concrete-to-abstract shift in development (Bruner, 1960; Piaget, 1952). This view is reflected in stage theory-based models of geometric development, such as the Van Hiele (1986) levels, which progress through visualization, analysis, abstraction, and (formal) rigor stages.

A potential pitfall of the instructional application of stage-based models is the imposition of artificial boundaries on student activities – e.g. unnecessarily delaying rigorous tasks for younger students or overlooking necessary visuo-spatial tasks for older students. In particular, research in the areas of perceptual learning (Gibson, 1969; Goldstone & Son, 2008, pp. 327–355) and grounded (embodied) cognition (Barsalou, 2008; Lakoff & Núñez, 2000, pp. xvii, 493) suggest that continued refinement of spatially-grounded representations of concepts is essential to guide higher-level knowledge (Black, Segal, Vitale, & Fadjo, 2012). Rather than shifting to abstraction, a grounded approach seeks to promote attention to increasingly precise visuo-spatial features of materials (Goldstone & Barsalou, 1998; Quinn & Eimas, 1997; Schyns, Goldstone, & Thibaut, 1998). As a geometric example, the concept of a square may be grounded in formally-relevant perceptual features – such as perpendicularity, bilateral and rotational symmetry, etc. – while learning to disregard other salient features – e.g. rotational orientation.

In their description of mathematical thinking, Lakoff and Núñez (2000, pp. xvii, 493) describe several mechanisms by which spatially-grounded knowledge develops. Specifically, “grounding metaphors” map spatial structures or actions to mathematical concepts. For example,

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