Computers & Education 62 (2013) 308–319

Contents lists available at SciVerse ScienceDirect

Computers & Education

journal homepage: www.elsevier.com/locate/compedu

Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving

Wu-Yuin Hwang [1], Shih-Shin Hu*

Graduate School of Network Learning Technology, National Central University, Jung-li, Taoyuan 300, Taiwan, ROC

ARTICLE INFO

Article history:
Received 27 December 2011
Received in revised form
4 October 2012
Accepted 14 October 2012

Keywords:
Collaborative learning
Elementary education
Interactive learning environments
Virtual reality

ABSTRACT

Learning geometry emphasizes the importance of exploring different representations such as virtual manipulatives, written math formulas, and verbal explanations, which help students build math concepts and develop critical thinking. Besides helping individuals construct math knowledge, peer interaction also plays a crucial role in promoting an understanding of mathematics and geometric problem solving. In this research, an Interactive Future Mathematics Classroom (IFMC) based on the Collaborative Virtual Reality Learning Environment (CVRLE) was proposed to facilitate three-dimensional (3-D) geometric problem solving. Two representational tools, the virtual manipulative and a white board, were integrated into the IFMC to help students with the following activities: to synchronously review peers' solving processes; to individually or collaboratively manipulate 3-D objects using the virtual manipulative; and to give comments on peers' white boards for future queries and discussions. One eight-week experiment was conducted and the results showed that the experimental group using the IFMC performed significantly better than the control group on geometric learning achievement. Further analysis showed that the peer learning behaviors of the experimental group in the two kinds of geometric problems, volume and surface area calculation, were different due to the problems' varying difficulty levels. Moreover, various peer learning behaviors with multiple representations lead to different types of strategies for geometric problem solving in the IFMC. Therefore, peer learning behaviors in the IFMC were found useful to facilitate geometric problem solving by sharing ideas and exploring multiple representations.

1. Introduction

Geometric education has students use geometry within the process of problem solving to understand and explain the physical world around them (Baki, Kosa, & Guven, 2009). To fully appreciate the real world and other fields such as computer graphics, engineering, and architecture, exploration of 3-D geometry (including moving, positioning, orienting, constructing, building, and communicating 3-D objects) is an important topic that should be exercised by students in a geometric learning curriculum (Yeh, 2007).

Some researchers conceived the nature of mathematics as the outcome of social process and math knowledge, which is thus understood to be fallible and eternally open to revision both in terms of its proofs and its concepts (Ernest, 1999). In addition to individual exploration of multiple representations for constructing geometric knowledge, which is urged by the development of constructivism, math reform gradually advocates that the interaction of sharing, assessing, and collaborating for discovering the solution with peers and their representations must be implemented in a student's problem solving process (NCTM, 2000; Wilkins, 2008; Wood, Williams, & McNeal, 2006). That is, students interact with the instructor, peers, and learning materials to share their thoughts and to verify solutions from multiple viewpoints. Obviously, communicating math concepts through mutual observation and discussion with peers often helps students identify unforeseen perceptions (Sheffield, 2000).

Due to the lack of examination of the impacts on students' three-dimensional (3-D) geometry concepts learning and problem solving caused by synchronous interaction with multiple representations among peers in the Collaborative Virtual Reality Learning Environment

* Corresponding author. Tel.: +886 3 4227151x35411; fax: +886 03 4261931.
E-mail addresses: wyhwang1206@gmail.com (W.-Y. Hwang), leaverij@gmail.com (S.-S. Hu).
[1] Tel.: +886 3 4227151x35411; fax: +886 03 4261931.

0360-1315/$ – see front matter
http://dx.doi.org/10.1016/j.compedu.2012.10.005

(CVRLE), this research proposed the Interactive Future Mathematics Classroom (IFMC), based on the CVRLE to facilitate students solving geometry problems and afterwards to study the effect of peer learning behaviors to learning achievement.

2. Literature review

2.1. Geometric problem solving with multiple representations

Goldin (1998) indicated that there were external representations in the real world and internal representations in the mind. Psychologists view the representation as the process of modeling concrete objects in the real world into abstract concepts or internal representations, which consist of mental components such as metaphorical, visual-spatial, and structural knowledge (Jonassen & Carr, 2000). For the mathematical context, Nakahara (2008) articulated five kinds of representations used in math education; the first representation is symbolic representation used in mathematical notions such as numbers, letters and symbols. The second is linguistic representation that is used every day in language such as Chinese and English. The third one is illustrative representation that uses figures, graphs, and so on. The fourth is manipulative representation such as teaching assistant tools that teachers or students can operate to figure out concepts or find clues. The fifth is realistic representation based on actual states and objects. Gagatsis and Shiakalli (2004) asserted that the diversity of representations for the same mathematical concepts, the connections between them, and the conversion from one representation to others were the keys to successfully learning math. Experiencing and exploring the functions of multiple representations have strongly proved to help students attain a deeper understanding of math concepts and the cognitive processes in problem solving (Debellis & Goldin, 2006; Hitt, 2002; Hwang, Su, Huang, & Dong, 2009). In terms of geometric learning, much research has employed the use of manipulative representations, supported by the virtual manipulative, and symbolic and illustrative representations, supported by the white board in geometric problem solving activities (Mithalal, 2010).

2.2. Virtual manipulative and 3-D geometry learning

Moyer, Bolyard, and Spikell (2002) defined the virtual manipulative as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge.” The use of manipulatives has proven helpful for assisting children in further developing of their concepts, procedures, and other aspects of mathematics (Hancock & Lane, 2010; Heddens, 2005; Munger, 2007). Currently, lots of virtual manipulatives referring to numeric operations, algebra, and geometry can be accessed via the Internet for individual use. For geometric learning, Dynamic Geometry Software (DGS) applications such as *Cabri-Geometry*, *Geometer's Sketchpad*, and *GeoGebra* (Hohenwarter & Preiner, 2007) have been widely adopted for use in school geometry curricula. Students utilize the DGS to manipulate and animate 2-D geometric shapes through “mouse dragging” actions. Besides plain geometry, dynamic 3-D software such as *Cabri3D* is used to explore 3-D geometric shapes such as cylinders, cones, and other primitives. Researchers have asserted that 3-D DGS is very suitable to quickly introduce students to 3-D geometry, while giving enough intuitive support to enable students to acquire a greater understanding of 3-D mathematical and spatial concepts (Accascina & Rogora, 2006; Baki et al., 2009; Christou, Jones, Mousoulides, & Pittalis, 2006).

Numerous studies have suggested constructivist learning of 3-D geometry, which argues that students should construct and realize geometric properties by manipulating 3-D objects (Hwang et al., 2009; Yeh, 2007). Furthermore, digital learning materials using computer visualization have been advocated to facilitate 3-D geometric learning. This suggestion has been made because digital learning materials can bridge the gap between the mental images of humankind and external representations around the world; this perception is viewed as the key to be able to successfully understand the process of solving geometric problems (Hauptman, 2010; McLeay, 2006). Therefore, learning 3-D geometry in CVRLEs where multiple representations could be constructed and demonstrated becomes promising.

2.3. White board and geometry learning

Many studies have used white boards to represent 3-D geometric knowledge as projected 2-D graphical and symbolic representations to help students understand abstract geometric properties. Hwang et al. (2009) proposed the VMW system to engage in geometric problem solving with a multimedia white board that allows students to express their mathematical ideas through writing texts, drawing pictures, and making annotations. The results showed that students were satisfied with the white board for improving their multi-representations transformation. Hung (2000) articulated that the sharing mechanism, and the facility of drawing tools provided by the white board, encourage students to “talk” and “write” their mathematical perceptions. Stahl, Wee, and Looi (2007) argued that the shared white board is necessary for supporting most geometry problems. They adopted a chat environment incorporating the white board with the “referencing” functionality that allows students to reference specific objects or areas in the drawing. Students therefore can collaboratively reach a deeper understanding of mathematical objects and their relationships through shared textual and graphical means. A shared white board also provides a flexible area to post drawings and text boxes that serve as knowledge artifacts for the group's memory. Cakir, Stahl, and Zemel (2010) claimed that using a white board is a representational practice that constructs shared mathematical diagrams to be interactive communications carried out for collaborative problem solving online. They also found that participants would find the solution to a problem gradually and recognize contexts through white board activities.

2.4. Collaborative geometry problem solving in virtual reality

CVRLEs such as Second Life have emerged as immersive and interactive learning environments for different areas in recent years. Due to the enhanced capabilities for 3-D visualization of various forms of representations, synchronous manipulation of objects and communication among multiple users, it has been suggested that CVRLEs are worthy of investigation as tools to help teach and learn 3-D geometric knowledge (Caprotti & Seppala, 2007; Hauptman, 2010). Besides individual representations, CVRLEs allow multiple representations created by peers for exploration. From the viewpoint of social constructivism, interpretations of mathematical meanings are shaped by experiences