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Literature Review

Computers in mathematics education – Training the mental number line

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ABSTRACT

Number magnitude is often described to be represented along a mental number line. In children, the accuracy of this mental number line seems to be associated with other basic numerical/arithmetic competencies. As a consequence, specific number line trainings have been developed. Evaluations indicated improvements not only in number line accuracy but also other numerical (e.g., magnitude comparison) and arithmetic tasks (e.g., mental addition) not trained directly. Importantly, this was observed for both board games as well as first computer-supported number line trainings. However, computer technology progresses rapidly. In this literature review, we specifically focus on the issues of multi-player learning environments and embodied interactions as new opportunities for training the mental number line. Regarding multi-player environments we discuss the adaptivity of learning environments needed to ensure balanced success rates in such trainings. As regards embodied interaction, we elaborate on new trainings allowing for bodily experiences of numerical concepts with new motion sensitive input devices and tangible user interfaces combining benefits of physical manipulatives with digitally provided symbolic information. We conclude that the latest developments in computer technology open up new directions for the training of the mental number line in particular and numerical/arithmetical competencies in general.

Contents

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1. Introduction

It is widely agreed that the spatial representation of numbers, often described by the metaphor of a mental number line, is one of the basic representations underlying successful number processing and arithmetic (Booth & Siegler, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003). Upon this mental number line, numbers are represented spatially in ascending order corresponding to their magnitude. First postulated back in 1970 by Restle, evidence for such a spatial representation of number magnitude has not only been observed in adults (e.g., Dehaene, Bossini, & Giraux, 1993). Rather, there is now accumulating evidence for systematic spatial–numerical associations in first-graders (van Galen & Reitsma, 2008), pre-counting children (e.g., Patro & Haman, 2012), and even pre-verbal infants (e.g., de Hevia & Spelke, 2010). Interestingly, the mental number line relates number magnitude and space with a specific spatial directionality. In western cultures with left-to-right reading and writing, smaller magnitudes are systematically associated with the left side and larger magnitudes with the right side of space (see Wood, Nuerk, Willmes, & Fischer, 2008 for a meta-analysis). Additionally, it was observed that even just looking at a number can induce spatial shifts of attention (Fischer, Warlop, Hill, & Fias, 2004) while the spatial direction of eye or head movements predicts number magnitudes produced in random number generation tasks (e.g., Loetscher, Schwarz, Schubiger, & Brugger, 2008). Thus, the association of numbers and space is automatic, bidirectional and present from early age on.

Importantly, the mental number line representation is associated with children's numerical competencies (e.g., Booth & Siegler, 2006; Geary, Hoard, Nugent, & Bailey, 2012). Therefore, the mental number line is a promising representation to be trained in numerical education and intervention. In children, the mental number line representation is usually assessed by the number line estimation task, in which participants have to estimate the location of a given target number (e.g., 17) on a presented number line (ranging, for example, from 0 to 100; Siegler & Opfer, 2003). While the spatial–numerical demands of this task are undisputed, the underlying processes are discussed more controversially. On the one hand, Moeller, Pixner, Kaufmann, Nuerk (2009); see also Moeller & Nuerk, 2011) argue that number line estimation is influenced by children's understanding of the base-10 place-value structure of the Arabic number system (i.e., their understanding of the composition of the Arabic number system in units, tens, etc.). On the other hand, Barth and Paladino (2011) suggest systematic influences of proportion judgment strategies on estimation performance. Thus, when training number line estimation, it is not only the spatial nature of the number magnitude representation which is practiced, but also other representations (e.g., place-value representation) or strategies (e.g., proportion judgment) which may be recruited to map numbers onto space. Interestingly, however, while there is a reliable association between number line estimation performance and arithmetic abilities, the correlation between the above described directional aspect of number-space associations and arithmetic capabilities has been controversially discussed (Cipora & Nuerk, 2013; Dehaene et al., 1993; Fischer & Rottmann, 2005). For the development and implementation of successful mental number line trainings aimed at fostering numerical development, training the continuous spatial character of the mental number line seems more promising than focusing on the directional aspect of the mental number line (see Patro, Nuerk, Cress, & Haman, 2014 for a discussion). Interestingly, this is also reflected in popular instructional materials for early mathematics education.

2. Linear numerical representations in mathematics education

When taking a look at popular instructional material and visual conceptualizations used in formal mathematics education in primary school, one cannot fail to notice that these predominantly refer to some type of linear format and/or arrangement very similar to the metaphor of a mental number line. For example, common manipulatives such as base-10 blocks (consisting of cubes as single units and rods of 10 cubes) incorporate linear formations of tens (see Fuson & Briars, 1990, for a study on the effects of these base-10 blocks in teaching of first- and second-graders). Similarly, ten-frame tiles or twenty-frame tiles (in which single items can be placed) are also arranged in linear formations (of e.g., 5 or 10) and are widely used to convey basic numerical knowledge (see e.g., Losq, 2005, for a study on the benefits of ten-frame tiles). Some educators have also used bead strings up to hundred, structured in segments of ten by alternating colors, to promote basic understanding of number magnitude.

A caveat all these materials and visual aids have in common is that it is possible for children to simply count the blocks, items or beads to produce a correct solution. While such counting strategies might be acceptable when children first enter school (e.g., Moeller, Martignon, Wessolowski, Engel & Nuerk, 2011a), they need to be overcome for children to transition to more effective calculation strategies. Moreover, the effectiveness of such manipulatives has been called into question, because it seems to depend heavily on teachers' competencies in using them. Therefore, some educators have introduced empty number lines to improve children's conceptualization of a certain number range and to illustrate the principles of arithmetic operations such as addition and subtraction. In this context, Kaufmann and Wessolowski (2014) argue that using such empty number lines can promote conceptual understanding and reduce the reliance on counting strategies. Empirically, Klein, Beishuizen, and Treffers (1998; see also Blöte, Van der Burg, & Klein, 2001) were able to promote second-graders competence in arithmetic operations with programs building on empty number lines.

In sum, linear representations and number lines are now quite common in early mathematics education. We will briefly describe in the following how simple board games have advanced research on number line trainings before we review and discuss computer-supported number line trainings.

3. Training the mental number line

In recent years, several number line interventions have been developed. On a quite basic level, Opfer and Siegler (2007) gave second-graders feedback on their number line estimations and found performance improvements that were "*strikingly abrupt, often occurring after a single feedback trial, and impressively broad, affecting estimates over the entire range of numbers from 0 to 1000*." (p. 169). Similarly, Siegler and Ramani (2008, see also Whyte & Bull, 2008) used linear board games, in which children had to move a playing piece along a linear and equidistant set of fields. The authors found that differences in number line estimations between low- and middle-income children were eliminated after just four 15 min sessions of training. In a series of follow-up studies, Ramani and Siegler (2008) observed positive transfer effects of such linear board games to children's performance in magnitude comparison, counting, and numeral identification, which were not observed for an almost identical color board game. Importantly, these transfer effects were not attributable to the per se use of numbers in the board games. Playing circular (instead of linear) number board games did not improve preschoolers'