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How best to teach the cardinality principle?

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ABSTRACT

The *cardinality principle* (CP), which specifies that the last number word used in the counting process indicates the total number of items in a collection, is a critically important aspect of numeracy. Only one published study has focused on how best to teach the CP, and its results are uncertain (Mix, Sandhofer, Moore, & Russell, 2012). The present study was designed to investigate several modeling procedure to teach the CP. Forty-nine 2–5-year olds were randomly assigned to one of the three interventions: (a) label and then count (label-first), (b) count with an emphasis on the last word and label (count-first), and (c) counting only. At a delayed posttest, the count-first intervention was substantively more efficacious than the other interventions at promoting success on the CP task and a transfer task (as measured by effect size). The results underscore the need for early childhood educators and parents to reinforce the purpose of counting by building on children's subitizing ability and explicitly labeling the total number of items *after* a collection is counted.

1. Introduction

Early number development provides a key basis for learning school mathematics (Sarama, 2007, 2008; Frye et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012; Sophian, 2004). Unfortunately, early deficiencies in numeracy can emerge because of the lack of learning opportunities, snowball, and seriously interfere with learning school mathematics (Baroody, Lai, & Mix, 2006; Dowker, 2005). For these reasons, there is currently considerable interest in early mathematical interventions to "level the playing field" (Frye et al., 2013).

1.1. Cardinality principle: a key foundation of numeracy

The cardinality principle (CP) entails understanding that the last number-word used in counting represents the total number of items in a collection. This concept, which typically develops between 3 and 5 years of age, is a developmental milestone that provides a foundation for early numeracy (Bermejo, 1996; Lago, 1990, 1994; Fuson, 1988; Gelman, Meck, & Merkin, 1986). For example, it underlies meaningful one-to-one object counting and enables children to respond meaningfully to the "how many" question when a collection is beyond their subitizing range. The CP appears to provide a critical role in generalizing insights (patterns and relations) gleaned from experiences with small, subitizeable (readily recognized) numbers of objects to larger numbers (Sarnecka & Carey, 2008). Children do not connect number words later in the counting sequence to quantity until they have developed the CP (Slusser & Sarnecka, 2011). Similarly, children who understand the CP (and only those children) appear to understand what it means for two collections to be exactly equal (Sarnecka & Wright, 2013). The CP also appears to be a developmental prerequisite for the successor principle ("adding exactly 1 object to a set means moving forward exactly 1 word in the list"; Sarnecka & Carey, 2008). Moreover, the CP underlies such advanced counting strategies as counting-on (e.g., starting a count from a number word such as "five" instead of "one") and informally adding-on (e.g., solving "5 and 3 more" by starting with the cardinal term "five" and counting "six, seven eight" instead of counting all; Fuson, 1988, 1992). For these reasons, the CP is widely cited as a key preschool and kindergarten goal (Alabama State Department of Education, 2015; Arizona Department of Education, 2016; Council of Chief State School Officers, 2010; Florida Department of Education, 2017; Frye et al., 2013; Georgia Department of Education, 2016; Illinois State Board of Education, 2013; North Carolina Department of Public Instruction, 2012; North Dakota Department of Public Instruction, 2011; Tennessee Department of Education, 2014; Texas State Board of Education, 2012).

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Even children who have mastered one-to-one counting may not understand the CP. As a result, when asked the how-many question, some children may misinterpret the question as a request to recount the collection. Others may learn that parroting back the last number word from a count seems to satisfy adults who ask the how-many question but may not realize that the purpose of counting is determining the total number of items in a collection. Fuson (1988) called such behavior a meaningless "last-number rule." Children may have difficulty learning the CP because of how counting and the principle are taught (Baroody et al., 2006). Unfortunately, adults often do not model the CP in a helpful manner (Mix, Sandhofer, Moore, & Russell, 2012). The purpose of the present study was to compare three different approaches for teaching this key numeracy concept.

1.2. Gaps in the research on teaching the cardinality principle

There is currently no published intervention research on the CP that provides clear guidelines on teaching the CP. In an attempt to examine how to best teach this concept, Mix et al. (2012) evaluated four instructional approaches: (a) counting alone without separately specifying the cardinal value or total (count-only), (b) specifying the total alone, (c) alternating between counting alone and labeling the total alone, and (d) labeling a collection with its cardinal value (total) first and then counting (labeling-first). These researchers hypothesized that only the last intervention would be efficacious in discovering the CP because the overlap between counting a collection and stating its cardinal value can signal that the two are linked (Bermejo, 1996; Carey, 2001; Klahr & Wallace, 1976; Schaeffer, Eggleston, & Scott, 1974; Spelke & Tsivkin, 2001). For instance, counting a collection of three items ("1, 2, 3") and naming it "three" creates an opportunity to notice that the last number word used in the counting process is the same as the collection's total (cardinal label). According to structure-mapping theory (Gentner, 2005), the overlap or commonality between counting and the cardinal label signals a connection between the two and may initiate a process of reflection that reveals the nature of the connection, namely the CP.

Indeed, of the four instructional methods, only the labeling-first training appeared effective in fostering the CP. Specifically, counting alone (without specifying the cardinal value or total), specifying the total alone, and alternating between counting alone and labeling the total alone were ineffective. Mix et al. (2012) also found that parents seldom used the label-first method when reading picture books.

However, Mix et al.'s (2012) results do *not* actually provide guidance on how best to teach the CP for two reasons. One is that the most common method for teaching the concept was not evaluated. The second reason is that an indirect and imprecise operational definition of the principle was used.

1.2.1. Instructional issue

A common method of teaching the CP is to model one-to-one counting, emphasize the last number word, and repeat the last number word (count-first method). For example, an adult might count a picture of five cookies by saying, "One, two, three, four, *f-i-v-e* (in a higher pitch)—see five cookies" (repeating the total). Mix et al. (2012) did not evaluate the count-first method because they reasoned that, as the counting process and the labeling process are not clearly separated, such a demonstration would be confusing or useless. They argued that labeling a collection first with a cardinal number (representing its total) and then counting the collection clearly disentangle the two processes. However, the counting-first method is also consistent with structure-mapping theory (Gentner, 2005) and so may be as efficacious as the labeling-first training. In fact, it may be more efficacious than Mix et al.'s modeling method because the last number word in a count and the cardinal value of a collection occur in greater proximity. In brief, it is an empirical question whether children can or cannot compare the two processes of the count-first procedure and whether the counting-first or the label-first method is more effective (Baroody & Purpura, 2017).

1.2.2. Methodological issue

The CP is a basic cardinality concept, which Fuson (1988, 1992) called the *count-cardinal concept*. Following the common practice in developmental psychology, Mix et al. (2012) assessed knowledge of the CP by using the give me-*n* task (e.g., asking a child to count out a collection of 6 items; see also Davidson, Eng, & Barner, 2012; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008; Sarnecka & Lee, 2009). However, a give-me-*n* task actually measures a cardinality concept more advanced than the CP—what Fuson (1988, 1992) called the cardinal-count concept (a cardinal number *specifies* the last number word used in producing or counting out a requested collection). Thus, the operational definition of the CP used by Mix et al. (2012) involved only an indirect measure of the count-cardinal concept or CP and may be confounded by task demands such as forgetting the requested amount (Baroody, Lai, & Mix, 2017; Baroody & Purpura, 2017).

In contrast, a how-many task is a direct measure of the CP, because it requires a child to count a collection and use the last number word in the process to identify the total number of items or cardinal value of a collection. Evidence indicates that in comparison to the how-many task and tasks that require the meaningful application of the CP, the give-me-*n* task appears to underestimate 3- and 4-year olds' CP knowledge (Baroody et al., 2017; Baroody & Purpura, 2017). It is plausible, then, that many of Mix et al.'s (2012) participants already knew the count-cardinal concept (CP) at pretest. Thus, the posttest gains in the study may have indicated improvement in the more advanced cardinal-count concept, NOT the basic count-cardinal concept (CP).

1.3. Rationale for the present study

The purpose of the present study was to compare the efficacy of the count-first method (a relatively common instructional approach not previously evaluated) with the labeling-first method (which Mix et al., 2012, found was successful) and the count-only method (which Mix et al., 2012, found was unsuccessful). The present study used a direct measure of the CP (the how-many task) to examine the efficacy of these instructional methods. Efficacy was also gauged using a transfer task, namely the give-me-*n* task, which serves to gauge the related but more advanced cardinal-count concept. The more basic count-cardinal concept is a developmental prerequisite of the cardinal-count concept (Fuson, 1988) and, thus, the give-me-*n* task assesses the meaningful application of the CP.

The study examined three hypotheses.

1.3.1. Hypothesis 1: The value of a commonality between counting and the cardinal label

Consistent with structure-mapping theory (Gentner, 2005), methods of modeling the CP that help connect counting with a cardinal outcome will be more successful in promoting understanding of the CP than counting alone on both the main (how-many) task (a direct measure of the CP) and the transfer (give-me-*n*) task (an indirect measure of the CP).

1.3.2. Hypothesis 2: The value of a close temporal connection

The count-first method will be more efficacious in promoting progress on both the main and transfer tasks than the labeling-first method because the former entails a closer temporal connection