International Journal of Educational Research 64 (2014) 92–106

Contents lists available at SciVerse ScienceDirect

International Journal of Educational Research

journal homepage: www.elsevier.com/locate/ijedures

Reasoning-and-proving in algebra: The case of two reform-oriented U.S. textbooks

Jon D. Davis [a,*], Dustin O. Smith [a], Abhik R. Roy [a], Yusuf K. Bilgic [b]

[a] Western Michigan University, United States
[b] State University of New York-Geneseo, United States

ARTICLE INFO

Article history:
Received 1 December 2012
Received in revised form 21 May 2013
Accepted 7 June 2013
Available online 1 August 2013

Keywords:
Reasoning-and-proving
Algebra
Textbook

ABSTRACT

This research study examined students' opportunities to engage in reasoning-and-proving (RP) within exposition and task components of two U.S. reform-oriented secondary algebra textbooks. There were statistically significant differences between the two textbooks in terms of the percentage of tasks coded as RP and statistically significant differences in the percentages of tasks devoted to RP across different algebra topic areas within each textbook. Differences also appeared in the role of technology in RP within both textbooks. While this study is focused on two U.S. algebra textbooks, broader recommendations will be made on textbook design with regard to RP. Moreover, the framework presented in this study provides researchers and teachers with tools to examine RP in textbooks and enacted classroom lessons.

1. Introduction and background

Researchers from around the world have recently been turning their focus on what we refer to as *proof-related constructs* (Davis, 2012; Hanna & de Bruyn, 1999; Stacey & Vincent, 2009; G. J. Stylianides, 2009; Thompson, Senk, & Johnson, 2012). This terminology will be used to refer to proof as well as its related constructs such as reasoning. The confluence of research in *proof-related constructs* in mathematics textbooks is important for two reasons. Proof is a principal component of the practice of mathematics (Hanna, 2007; Schoenfeld, 2009; Weber, 2008) while textbooks are a principal component of the practice of teaching mathematics (Grouws & Smith, 2000; Tarr et al., 2008; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). The study described here expands a framework developed by G. J. Stylianides, thus providing researchers and teachers with additional tools with which analyses of reasoning-and-proving within the written curriculum can be conducted. Moreover, the results have implications for the design of textbooks that seek to incorporate RP.

This review is organized around six different themes. This review begins with a description of how secondary mathematics textbooks in the U.S. have been categorized in order to orient readers to issues with regard to curriculum. As this study involved analyses of the narrative portions of the textbook as well as student tasks the review describes previous research involving proof-related constructs within these two areas. The review continues with a description of research involving students' proof engagement within and across textbooks. Next, the review discusses the connectedness among different proof-related constructs. Last, the review includes a section on technological tools in proof-related constructs.

* Corresponding author. Tel.: +1 269 387 4591; fax: +1 269 387 4530.
E-mail address: jon.davis@wmich.edu (J.D. Davis).

0883-0355/$ – see front matter
http://dx.doi.org/10.1016/j.ijer.2013.06.012

1.1. Categorizing mathematics curricula in the United States

Mathematics curricula in the United States have been categorized as either reform-oriented or conventional. The first category refers to curricula that relied on national reform documents such as the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) as design templates. The second category refers to textbooks that existed at the time of the development of reform-oriented curricula or have been created by publishers through the assistance of editors and an invited set of authors. Conventional curricula used a back mapping process to match their curricular content and processes to national reform documents.

1.2. Opportunities for students to engage in proof-related constructs in textbook tasks

Studies in the United States (Davis, 2012; G. J. Stylianides, 2009; Thompson et al., 2012) and other countries (Nordström & Löfwall, 2005) highlight the limited number of student tasks related to proof in school mathematics textbooks. In the U.S., Thompson et al. found that 5.4% of 9742 student tasks across 22 textbooks in the areas of exponent properties, logarithm properties, and polynomials engage students in *proof-related reasoning*.[1] Nordström and Löfwall examined the prevalence of proof in five different courses in upper secondary school mathematics in Sweden covering the following content areas: algebra; geometry; statistics and probability; functions and calculus; exponents and logarithms; trigonometry; and complex numbers. They found that about 2% of the tasks in these content areas asked students to construct proofs. G. J. Stylianides' examination of a middle school (ages 11–14) reform-oriented textbook series revealed that 5% of 4578 tasks asked students to construct what he described as demonstrations.[2]

An exception to these studies is work conducted by Hanna and de Bruyn (1999). They used a framework consisting of three categories: proof, discussion of proof, and non-proof. The second category involves discussions about the creation of a proof or directions about how to develop a proof. Proof items consisted of full or partial proofs. They found that 21% of 1086 problems in one textbook involved the construction or discussion of proof.

1.3. Opportunities for students to read about proof-related constructs in student textbook narratives

Hanna and de Bruyn (1999) found that 22% of 465 narrative items in one popular 12th grade mathematics textbook and 17% of 621 narrative items in another popular 12th grade mathematics textbook consisted of proof or the discussion of proof. Stacey and Vincent (2009) examined the modes of reasoning used in the narrative sections of seven different topics appearing in nine textbooks for eighth grade Australian students. Modes of reasoning are related to Harel and Sowder's (1998) proof schemes yet they differ from this construct in the following way. A proof scheme consists of the method by which a person or community removes doubt about the validity of an assertion. Modes of reasoning are presented in the textbook and are not necessarily connected to a particular human agent as the textbook authors may have chosen that explanation for purely pedagogical reasons. Stacey and Vincent found a total of seven modes of reasoning within seven different content areas in the different textbooks: deduction using a general case; deduction using a specific case; deduction using a model; concordance of a rule with a model; experimental demonstration; appeal to authority; and qualitative analogy. In their analysis of exponent, logarithm, and polynomial properties in 22 secondary mathematics textbooks, Thompson et al. (2012) found that 69% of properties appearing in the narrative sections were justified in a variety of different ways, but 60% of this number contained valid proofs. Consequently, less than half of the properties appearing in the textbooks were justified with a valid mathematical proof. They also found that 60% of the statements with no justification were accompanied by specific examples showing how to apply the property suggesting that examples might constitute a suitable justification for the validity of a property.

1.4. Proof-related constructs varies within and across textbooks

Researchers have also sought to compare and contrast the presence of proof across textbooks for school mathematics students within a certain age group. Among 22 different secondary school (ages 14–18) mathematics textbooks in the U.S., Thompson et al. (2012) found that 14.7% of 532 student tasks involved proof-related reasoning in a reform-oriented mathematics program. On the other hand, 3.7% of 2042 tasks involved proof-related reasoning in a conventional textbook series. In a similar vein, Davis (2012) found that 22% of 1158 tasks in a reform-oriented textbook unit were coded as reasoning-and-proving[3] (RP), but comparatively fewer student tasks in the conventional textbook unit were considered to be RP (5% of 1129). Hanna and de Bruyn (1999) found that 21% of 1086 problems in the *Foundations of Mathematics 12* textbook and 16% of 1491 problems in *Mathematics 12* involved proof or the discussion of proof. Stacey and Vincent (2009) found that

[1] Thompson et al. (2012) refer to proof-related reasoning as consisting of one or more of the following activities: finding a counterexample, making a conjecture, investigating a conjecture, developing an argument, evaluating an argument, and correcting a mistake in an argument.

[2] G. J. Stylianides (2008) defined a demonstration as a proof or "valid argument based on accepted truths for or against a mathematical claim" (p. 11) that do not depend "on the 'representativeness' of a particular case" (pp. 11-12).

[3] Davis (in press) used a framework constructed from the work of G. J. Stylianides (2009) who emphasized the integrated nature of reasoning-and-proving as consisting of identifying patterns, constructing conjectures, develop non-proof arguments, and creating proofs.