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The introduction of proof in secondary geometry textbooks[☆]

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ABSTRACT

Explicit reasoning-and-proving opportunities in the United States are often relegated to a single secondary geometry course. This study analyzed the reasoning-and-proving opportunities in six U.S. geometry textbooks, giving particular attention to the chapter that introduced proof. Analysis focused on the types of reasoning-and-proving activities expected of students and the type of mathematical statement around which the reasoning-and-proving took place, be it general or particular. Results include the fact that reasoning-and-proving opportunities in student exercises were predominantly of the particular type, whereas textbook exposition most commonly had general statements. Within the chapters introducing proof, opportunities for students to develop proofs were less common than exercises involving conjectures and statements or exercises about the reasoning-and-proving process. Opportunities to reflect on the reasoning-and-proving process were prevalent in the introduction chapters, though rare in the remainder of the books.

1. Introduction

Proof has a central role in the discipline of mathematics (Hanna, 2000; Hersh, 2009) and represents, for students of school mathematics, a formal culmination of the processes of sense-making and justification (National Council of Teachers of Mathematics [NCTM], 2009) that, ideally, began at an early age. Moreover, proof and justification processes in school mathematics can not only function to explain and verify but also as a means for fostering valuable skills and dispositions in students and providing formative assessment information for teachers (Staples, Bartlo, & Thanheiser, 2012). Because of these beneficial roles of proof, efforts have been undertaken in different parts of the world to integrate proving more fully into school curricula. In the United States, for example, NCTM has made recent efforts through policy publications and teacher professional development efforts (NCTM, 2009, 2011) to promote reasoning and sense-making for students of all ages. They point out that the exact form reasoning and sense-making takes depends upon classroom contexts and varies along a continuum from informal observations and explanations to formal conjectures and logical arguments, with mathematical proof constituting the formal endpoint of reasoning. The hyphenated term "reasoning-and-proving" (Stylianides, 2009) has arisen to capture the fact that the process of proving is much larger than proof itself, consisting of empirical explorations, conjecturing, justifying, refining, explaining, and so forth. With this broad notion of reasoning-and-proving, researchers have been able to examine reasoning-and-proving practices in various grade levels (e.g., Bieda, 2010; Stylianides, 2007; Stylianou, Blanton, & Knuth, 2009).

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Because reasoning-and-proving is such a broad practice, however, it is possible that students do not recognize their informal reasoning experiences as being related to proof. For example, students who explain how they know their solution is correct may not realize that this explanation could essentially prove their result. Evidence of such disconnects has been found in studies where students had limited conceptions of the role of proof in mathematics (Herbst & Brach, 2006; Schoenfeld, 1988; Soucy McCrone & Martin, 2009). This potential disconnect between broad reasoning-and-proving processes and those activities under the specific label of proof points to the importance of the way in which mathematical proof is explicitly introduced to students. Is proof introduced in ways that connect to students' past experiences, allowing them to reflect on how their previous processes of sense-making, justification, and explanation relate to the more formal process of proof? Does the introduction of proof highlight how all of these processes can be powerful sources of understanding (de Villiers, 1995; Hanna, 1990) and lay the foundation for students' future experiences with reasoning-and-proving?

One of the most important aspects of this foundation for students' future experiences is that students are guided to see the intellectual necessity of transitioning from more informal ways of reasoning to formal mathematical proof, and in particular, deductive argumentation. Harel and Tall (1991) articulated the *necessity principle* as a way to think about this issue in general. The necessity principle states that subject matter should be presented in ways that encourage learners to see its intellectual necessity, "[f]or if students do not see the rationale for an idea, the idea would seem to them as being evoked arbitrarily; it does not become a concept *of the students*" (p. 41, emphasis in original). There is evidence, however, that students in many countries often fail to see the intellectual impetus behind proof in mathematics (Chazan, 1993; Fujita, Jones, & Kunimune, 2009; Harel & Sowder, 2007; Porteous, 1990; Soucy McCrone & Martin, 2009).

The explicit introduction of proof typically occurs in a secondary-level geometry course, both in the United States (Herbst, 2002) and elsewhere (Hanna & de Bruyn, 1999; Jones, Fujita, Clarke, & Lu, 2008). This study examines the chapters in geometry textbooks that introduce proof and gives particular attention to the ways in which the reasoning-and-proving opportunities might support students in seeing the necessity of deductive forms of reasoning. Details about how we have operationalized this dimension of analysis are contained in Section 2.2. In a past study (Otten, Gilbertson, Males, & Clark, 2011), we characterized in aggregate the reasoning-and-proving opportunities in secondary-level geometry textbooks. Here, we focus on the following research questions:

1. What is the nature of reasoning-and-proving opportunities in the textbook chapters that introduce proof?
2. How do the reasoning-and-proving opportunities in these chapters compare to the reasoning-and-proving opportunities in the remainder of the textbooks?
3. What opportunities do the chapters that introduce proof provide for students to make reasoning-and-proving an object of attention or reflection?

The site for this study is student editions of geometry textbooks published for use in the United States. Although we analyzed United States textbooks, the issues raised are international in scope. In any nation, it is worthwhile to carefully consider the ways in which a process such as reasoning-and-proving, which is integral to what it means to do mathematics, is treated during the explicit shift toward deductive reasoning. Even if other textbooks do not have all the same characteristics as those in the present study, much may still be learned by comparing and contrasting other textbooks with those here and clarifying where the differences lie and why they might be important. With that being said, there are also known similarities with respect to reasoning-and-proving between textbooks from different countries. For example, similar to arguments made about textbooks in the United States, Fujita et al. (2009) found that Japanese textbooks treat proof in a way that does not "illustrate convincingly for students the difference between formal proof and experimental verification" (p. 176).

As Stylianides argued in the editorial of this issue, curriculum materials are an important factor in students' educational experiences (Grouws, Smith, & Sztajn, 2004; McCrory, Francis, & Young, 2008; Ni & Cai, 2011), and although teachers report substantial reliance on mathematics textbooks (Banilower et al., 2013), we recognize that teachers mediate the influence of those materials in important ways (Cohen, Raudenbush, & Ball, 2003; Tarr, Chavez, Reys, & Reys, 2006). With regard to reasoning-and-proving in textbooks, several past studies have focused on students' opportunities to engage in reasoning-and-proving activities, such as conjecturing, finding a counterexample, or proving (e.g., Davis, 2010; Stylianides, 2009; Thompson, Senk, & Johnson, 2012). These studies have documented that reasoning-and-proving opportunities outside of geometry are limited and insufficiently robust in guiding students to develop deep understanding of reasoning-and-proving, unless teachers fill in substantial gaps.

2. Methods

2.1. Textbook sample

This study involved six U.S. textbooks (see Table 1) designed for stand-alone geometry courses at the secondary level (i.e., texts meant to be used for a course covering geometry content only, as opposed to an integrated mathematics course), for students 13–16 years old. The textbook series that include these geometry texts constitute the mathematics textbooks used by approximately 90% of the U.S. secondary student population (Dossey, Halvorsen, & Soucy McCrone, 2008; Banilower et al., 2013).