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Reasoning-and-proving in mathematics textbooks for prospective elementary teachers

Raven McCrory [a,*], Andreas J. Stylianides [b,1]

[a] Michigan State University, Department of Teacher Education, 620 Farm Lane #114, Erickson Hall, East Lansing, MI 48824, USA
[b] University of Cambridge, Faculty of Education, 184 Hills Road, Cambridge CB2 8PQ, UK

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ABSTRACT

In the United States, elementary teachers (grades 1–5 or 6, ages 6–11 years) typically have weak knowledge of reasoning-and-proving, and may have few opportunities to learn about this important activity after they complete their teacher education program. In this study we explored how reasoning-and-proving is treated in the 16 extant textbooks written for mathematics courses for future elementary teachers in the United States to offer insight into the opportunities designed for them to develop knowledge about reasoning-and-proving. Our findings suggest that reasoning-and-proving is rarely addressed explicitly in these textbooks. Although many textbooks have one section or several sections in a single chapter (often the opening chapter) with content about reasoning-and-proving, references to reasoning-and-proving concepts and methods are rare outside of these few sections. We discuss methodological issues in studying the treatment of reasoning-and-proving in textbooks for teachers and implications of our findings for future research and teacher education.

1. Introduction

Over the past decade there has been increased recognition of the importance of reasoning-and-proving in students' mathematical education.[2] For example, in the United States, the setting of this paper, policy documents that set curriculum standards for school mathematics (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) have made calls for reasoning-and-proving to become central to all students' mathematical experiences and across all school years.

One reason for the increased emphasis on reasoning-and-proving in school mathematics is its key role in mathematical sense making (e.g., Ball & Bass, 2003; Hanna, 2000; NCTM, 2000). This is consistent with current efforts to organize classrooms as communities of mathematical discourse in which the validity of ideas rests on reason and argument (e.g., Carpenter, Franke, & Levi, 2003; Lampert, 2001). Although most examples of classrooms organized in

* Corresponding author. Tel.: +1 517 353 8565.
E-mail addresses: mccrory@msu.edu (R. McCrory), as899@cam.ac.uk (A.J. Stylianides).
[1] Tel.: +44 01223 767550.

[2] In this paper, we follow Stylianides (2008) in using the hyphenated term *reasoning-and-proving* to describe the overarching mathematical activity that encompasses the family of activities broadly related to generating and justifying mathematical generalizations, such as identifying patterns, making conjectures, and constructing arguments to prove or refute conjectures.

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this way have been taught by exemplary teachers, reasoning-and-proving can be important in all mathematics classrooms, regardless of instructional tradition, as long as students are expected to engage in exploring the truth of mathematical assertions. These assertions may derive from the textbook, the teacher, or students themselves.

Another reason for the increased emphasis on reasoning-and-proving is recognition of the value for children to engage, in developmentally appropriate ways, with practices that are honest to what it means to do mathematics in the discipline (e.g., Lampert, 2001; Stein & Smith, 2011; Stylianides, 2007a). This finds support from the work of educational thinkers such as Bruner (1960) who argued that there should be continuity between what a scholar does on the forefront of a discipline and what a child does in approaching the discipline for the first time.

The importance of reasoning-and-proving in school mathematics raises, then, issues about opportunities offered to prospective elementary teachers, in university mathematics courses required for teacher preparation, to learn about this mathematical activity. Investigating these opportunities is crucial in light of (1) the difficulties that many elementary teachers face with reasoning-and-proving (e.g., Martin & Harel, 1989; Simon & Blume, 1996); (2) the research finding that students' opportunities to learn are dependent on the quality of their teachers' knowledge (e.g., Hill, Rowan, & Ball, 2005); and (3) the recent research findings about teacher guides that offer inadequate support to teachers for supporting their students' learning of reasoning-and-proving (Stylianides, 2007b).

In this paper, we contribute to this body of research and explore the following question:

> In textbooks for undergraduate mathematics courses for prospective elementary teachers in the United States, what opportunities are designed for them to learn about reasoning-and-proving?

Our purpose is to understand, from the perspective of the future teachers and their instructors, what a textbook can provide about reasoning-and-proving. We use the term "designed" and say "can provide" to emphasize that, no matter what the content of the textbook is, it does not determine what students learn or what the instructor teaches. The textbook is a resource for, but not a determinant of, student learning. We take the perspective of seeking to explore what students could learn from the textbook if they used it for studying issues and concepts of reasoning-and-proving, and what instructors might teach if they based their course primarily on the textbook. Our assumption is that these two overlap to the extent that both students and instructors rely on the textbook for defining the course.

Although we focus on textbooks written for use in the United States, the methodological issues we faced would likely apply to similar textbook analyses in other countries. Also, the issues raised by the findings of our examination can offer a basis for reflection on relevant issues in other countries where (to the best of our knowledge) no similar analyses have been conducted.

2. Method

The data for this paper derive from analysis of 16 current mathematics textbooks in print in the U.S., for prospective elementary (grades 1–5 or 6, ages 6–11 years) teachers in the U.S. (a list of these textbooks is in Appendix A). The first step in our analysis was to decide how to locate reasoning-and-proving in these texts. Unlike topics such as congruence or regular polygons, reasoning-and-proving is a mathematical activity that spans mathematical topics and may appear in multiple places in textbooks. Taking the perspective of the students and instructors as users of the textbook, we focused our analysis on locations in textbooks where authors *explicitly* addressed issues related to reasoning-and-proving. That is, we sought reasoning-and-proving in the textbook without reading every chapter and section, imagining a student who wants to recall a specific idea mentioned in class or study for a test (asking a question such as, "what does it mean to do an indirect proof?"), or an instructor who is basing the sequencing and flow of the course on the textbook. This turned our attention to the published maps of each textbook; specifically, tables of contents and indexes.

This approach supports a consistent analysis of textbooks. In addition, it allows us to take the perspective of the student aiming to study reasoning-and-proving using the textbook, and of an instructor planning a course and interested in focusing on reasoning-and-proving. In each case, these users might look for explicit references to reasoning-and-proving within the textbook. Without such links, a student would need to search page by page, or remember something from earlier uses of the textbook. Similarly, an instructor who aimed to prepare lessons building a coherent approach to reasoning-and-proving while using the textbook as a primary source might also rely on the author's pointers to reasoning-and-proving as for content, organization, and trajectory. In short, the table of contents and index of a textbook give users a means for studying, learning, or teaching particular content. The disadvantage of this approach is that it may miss some instances of reasoning-and-proving that are not referenced in either the table of contents or the index. Since this limitation reproduces the problem a student or an instructor would have, our method is legitimate for our purposes.

To begin this process, we developed a list of terms that would be logical indicators of reasoning-and-proving. These terms related broadly to the two major activities comprising reasoning-and-proving (Stylianides, 2008), namely, *generating mathematical generalizations* and *justifying mathematical generalizations*. Under generating mathematical generalizations we had the terms *conjecture, generalization*, and *inductive reasoning* (or simply *reasoning*). Under justifying mathematical generalizations we had: (1) terms that relate to the development of mathematical arguments: