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Reasoning-and-proving in the written curriculum: Lessons and implications for teachers, curriculum designers, and researchers

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ABSTRACT

Reasoning-and-proving is fundamental to mathematics. The opportunities provided within the written curriculum of textbooks for students to engage with this fundamental process have the potential to influence students' mathematical learning in significant ways. The five papers in this special issue shed light on the opportunities within the curriculum related to reasoning-and-proving from the elementary level to the university undergraduate level. Together, they offer lessons and implications for teachers, curriculum designers, and researchers.

1. Introduction

The papers in this special issue have used the lens of curriculum analysis to analyze the important mathematical process of reasoning-and-proving, using the view that this process includes precursor activities such as identifying patterns and generating conjectures as well as developing formal proofs (Stylianides, 2008). Curriculum analysis can occur at many levels: the *intended* curriculum of state or national standards; the *written* curriculum of the textbook; the *enacted* or *implemented* curriculum of the classroom; the *assessed* curriculum of teacher, state, or national exams; and the *attained* curriculum of student achievement (Burkhardt, Fraser, & Ridgway, 1990; Hirsch, Lappan, Reys, & Reys, 2005; Stein, Remillard, & Smith, 2007; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Each level of curriculum provides different insight into influences relative to mathematics teaching and to students' opportunity to learn mathematics. For instance, Reys and colleagues (Reys, 2006) investigated the K-8 mathematics standards across U.S. states to document the variability in those standards and the different expectations for mathematics learning at various grade levels within the states.[1] Numerous researchers (e.g., Remillard, Herbel-Eisenmann, & Lloyd, 2009; Tarr, Chávez, Reys R.E, & Reys B.J., 2006; Thompson & Senk, 2010) have documented differences in how teachers enact curriculum as part of their instruction, often noting challenges teachers face implementing newer or alternative curricula. Hunsader, Thompson, and Zorin (2013) analyzed the extent to which mathematical processes are evident within the assessments accompanying published elementary grades curricula, finding that many unit assessments provide relatively limited opportunities for students to engage with the mathematical processes of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) with the exception of connections to real-world contexts. Other researchers have investigated what students have attained when studying from

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[1] The finalization in 2010 of the Common Core State Standards for Mathematics (Council of Chief State School Officers, 2010) and the subsequent adoption by more than 40 U.S. states and territories has been an attempt to alleviate some of this variability in mathematics expectations at different grade levels by providing a set of national standards.

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different curricula (e.g., Grouws, Tarr, Chávez, Sears, Soria, & Taylan, 2013; Senk & Thompson, 2003; Tarr, Reys, Reys, Chávez, Shih, & Osterlind, 2008).

One might wonder about the benefit of simply analyzing the written curriculum of textbooks without considering how they are used by teachers or what students learn. Yet, textbooks are a bridge between the intended curriculum of standards and their enactment in the classroom; they provide one instantiation for teachers of how the standards might be interpreted and what policy makers expect students to learn (Valverde et al., 2002). As noted by McKnight et al. (1987) and Schmidt et al. (2001), textbooks link the intentions of standards to the mathematical attainment of students.

From studies in various countries, we know that teachers use their textbook and accompanying resources to determine content and instruction (e.g., Blok, Otter, & Roeleveld, 2002; Grouws, Smith, & Sztajn, 2004; Sekiguchi, 2006; Yoshikawa, 2008), reinforcing the notion that "most student learning is directed by the text rather than the teacher" (Begle, 1973, p. 209). Indeed, Begle noted that the textbook is "the only variable that on the one hand we can manipulate and on the other hand does affect student learning" (p. 209). Given the critical role that textbooks play in influencing students' opportunity to learn, it seems that a first step to enhance students' opportunities and ultimate achievement is to have a clear understanding of what exists in the textbooks. Then, teachers, curriculum developers, and researchers can make necessary changes to this main driver of classroom instruction in order to improve the teaching and learning cycle.

The five original papers in this special issue provide valuable insights into conducting analysis of the written curriculum. Their combined focus on reasoning-and-proving in the written curriculum, together with that of other researchers (e.g., Stylianides, 2009; Thompson, Senk, & Johnson, 2012), has implications for teachers, curriculum developers, and researchers.

2. Issues addressed

In order to conduct their analyses, each of the teams represented by the five papers had to address a number of potential research issues, and find solutions that were meaningful in terms of the research questions and practical in their implementation. In the following sections, I discuss several of these issues.

2.1. The difficulty of identifying opportunities to engage with processes

As noted in the *Principles and Standards for School Mathematics*, reasoning-and-proving is fundamental to mathematics and should be incorporated into the mathematics curriculum K-12 (NCTM, 2000). But as noted by McCrory and Stylianides (2014), it can be difficult to locate where reasoning-and-proving is found within a textbook. Content topics are relatively easy to find because they are typically *explicit* in the curriculum and listed in Tables of Contents, lesson titles, and indices. In contrast, mathematical processes are *implicit* in the curriculum; in essence, the processes might be considered the glue that holds and weaves different aspects of content together. Student engagement with the processes is subtle and often hidden, being a function of the expectations students need to bring to the table as they work on problems and the expectations that teachers have about the work that students will present.

Each author team represented in this issue had to develop a means to find and then analyze these implicit references to mathematical reasoning-and-proving. There are lessons to be learned from their varied approaches, particularly because the process, as represented by the hyphenated term *reasoning-and-proving*, involves more than just locating a formal written proof.

To some extent, each paper made reference to a common framework as the basis for their analysis, namely the work of Stylianides (2008, 2009) who is serving as editor of this special issue. One advantage of this choice by the teams is that researchers are able to determine the extent to which a given framework for analysis can be applied across textbooks for different grade levels and with different purposes – from elementary textbooks for students and teachers designed to focus on many content topics to geometry textbooks for high school students with a major focus on proof. For the research community, it is helpful to have analytic tools that can be widely used rather than having many narrowly focused tools.

In addition, because the author teams had a common framework as the basis for their own analytic frameworks, there are common definitions for components of reasoning-and-proving. For instance, each looked at making and justifying conjectures and various types of proof and non-proof arguments. This enables readers to consider similarities and differences in the results from the various studies. Teachers can look at the results across levels to determine how experiences at one level can and should build on each other. Curriculum developers can consider how the expectations in their materials should expand from year to year. Researchers can design studies to determine what students have learned at one level that should be prerequisite knowledge at other levels, so that follow-up studies can focus on enhanced learning or what remediation might be needed.

The author teams used various strategies to ferret out instances of reasoning-and-proving within the many lessons of a textbook. Any research endeavor requires time, but time is money and this is typically a limited resource. So, an issue is how best to conduct the research to ensure the results are both reliable and useful. Clearly, every word of every textbook could be read and analyzed, but such an approach is not usually feasible. So, how might researchers sample the text? Bieda, Ji, Drwencke, and Picard (2014) analyzed every other lesson, looking at narratives, exercises, extensions, and assessments. Davis, Smith, Roy, and Bilgic (2014) modified this approach slightly by randomly selecting lessons from each chapter, with the number of lessons selected per chapter based on the length of the chapter. Both approaches resulted in sampling about half of the lessons in the text.