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Evaluation of three interventions teaching area measurement as spatial structuring to young children☆

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A B S T R A C T

We evaluated the effects of three instructional interventions designed to support young children's understanding of area measurement as a structuring process. Replicating microgenetic procedures we used in previous research with older children to ascertain whether we can build these competencies earlier, we also extended the previous focus on correctness to include analyses of children's use of procedural and conceptual knowledge and examined individual differences in strategy shifts before and after transitions, enabling a more detailed examination of the hypothesized necessity of development through each level of a learning trajectory. The two experimental interventions focused on a dynamic conception of area measurement while also emphasizing unit concepts, such as unit identification, iteration, and composition. The findings confirm and extend earlier results that seeing a complete record of the structure of the 2D array—in the form of a drawing of organized rows and columns—supported children's spatial structuring and performance.

1. Introduction

Measurement of area is not only an important mathematical topic, but also connects to children's experience with other mathematical topics and with the physical world. For example, developing concepts and procedures in area measurement builds a foundation for understanding topics such as multiplication, fractions, and composition of geometric figures, as well as for addressing real-world problems such as deforestation, navigation, designing of packing material, or measuring of molecules on a surface with a nanomeasuring machine. In this study, we replicated and substantially extended microgenetic procedures we used previously with students in Grades 2–5 to evaluate the effects of experimental interventions designed to support children's understanding of area measurement with spatial structuring on children from Grades 1–3.

2. Background

Students in the United States often use rote procedures for area measurement without demonstrating understanding of crucial

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area concepts and principles. Over 40 years of studies show that U.S. students, including young children, have performed poorly on measurement assessments, including area measurement (Battista & Clements, 1996; Carpenter, Coburn, Reys, & Wilson, 1975; Clements & Battista, 2001; Hirstein, 1981). As specific examples, on the fourth National Assessment for Educational Progress (NAEP) assessment, only 46% of seventh graders correctly identified the area of a 5 by 6 rectangle (Lindquist & Kouba, 1989). The trend in student performance on area does not seem to be improving. On the 2011 NAEP, only 24% of fourth graders were able to choose the correct value for the area of a square, and overall, measurement was the weakest of five substrands for fourth and eighth graders (National Assessment of Educational Progress, 2013).

Similar difficulties with area are revealed with younger children as well. A study of children from Grades 1, 2, and 3 revealed little understanding of area measurement (Lehrer, Jenkins, & Osana, 1998). When asked how much space a square (and a triangle) covers, 41% of children used a ruler to measure length and gave a numerical response, such as “9.” When asked what “9” would mean in that context, they said it would be “inches.” The second most frequent response, 22%, was “I don’t know.”

In investigating why children may struggle with area measurement, researchers have analyzed instructional textbooks and manipulative materials (Smith, Males, Dietiker, Lee, & Mosier, 2013), as well as common errors and misconceptions demonstrated by children. In presenting area tasks, textbooks often provide regions already subdivided or partitioned, so children simply need to count the number of squares, often counting one by one, to determine the area (Cavanagh, 2008). Representations such as this limit students’ experience with area measurement to simple counting of pre-formed units and later to using formulas without adequate understanding of constraints or adjustments to context. This results in incomplete understandings of fundamental area concepts, including understanding the attribute of area, unit concepts, accumulation and additivity, conservation, and the relation between number and space (Lehrer, 2003; Lehrer, Jacobson et al., 1998; Sarama & Clements, 2009).

A recent review supports the position that curricula have systemic deficits in teaching area measurement (Smith, Males, & Gonulates, 2016). Each of three elementary textbooks presented area measurement in almost exclusively procedural terms. Conceptual principles were infrequently provided and usually only after the procedures. A striking weakness was a lack of support for understanding how the multiplication of lengths produces area measures.

We postulate that addressing such weaknesses and prompting better understanding of area should be based on students’ development of spatial structuring competencies regarding organizing two- or three-dimensional space into orthogonal units. That is, spatial structuring is the mental operation of constructing an organization or form for an object or set of objects in space; it is a form of abstraction which involves the process of selecting, coordinating, unifying, and registering in memory a set of mental objects and actions. Based on Piaget and Inhelder’s (Piaget & Inhelder, 1967; Piaget, Inhelder, & Szeminska, 1948/1960) original formulation of coordinating dimensions, and observations by Outhred and Mitchelmore (1992), spatial structuring takes previously abstracted items as content and integrates them to form new structures, such as using a unit square to compose a row of units. It creates stable patterns of mental actions that an individual uses to link sensory experiences, rather than the sensory input of the experiences themselves, for example, selecting, coordinating, unifying, and registering in memory a set of mental objects and actions so that the row of units becomes a unit that can itself be iterated. Research indicates that such spatial structuring precedes meaningful mathematical use of the structures, such as determining area (Battista & Clements, 1996; Battista, Clements, Arnoff, Battista, & Borrow, 1998; Outhred & Mitchelmore, 1992). The scheme of spatial structuring in this context involves recognition of the goal of partitioning a region into parts, and the activity of organizing the region into a row-and-column structure, with the result of a fully partitioned and quantifiable region. Ultimately the structure is related to the linear dimensions of the rectangular region.

3. Theoretical framework

3.1. Hierarchic interactionalism

Learning trajectories (Simon, 1995) have served as the core of multiple research projects, curricula, and professional development projects (e.g., Clements & Sarama, 2014; Confrey, Maloney, Nguyen, & Rupp, 2014; Wilson, 2014). We define learning trajectories (LTs) as developmental progressions that include descriptions of children’s thinking and learning, as well as a related, conjectured route through a set of instructional activities to achieve a mathematical competence (Clements & Sarama, 2004).

We view LTs and children’s development of geometric measurement understanding through a theoretical lens termed *Hierarchic Interactionalism* (Sarama & Clements, 2009). A main tenet is that children progress through levels of understanding for measurement in ways that can be characterized by specific mental objects and actions (i.e., both concept and process) with the most visible progress through levels for domain-specific topics, such as area measurement. We postulate that various models and types of thinking grow in tandem to a degree, but a critical mass of ideas from each level must be constructed before thinking characteristic of the subsequent level becomes ascendant in the child’s thinking and behavior (Clements, 1992; Clements, Battista, & Sarama, 2001). This often involves “fallback” to prior levels of thinking under increasingly complex demands. Further, intense “experiences can engender rapid change to a new level,” even progressing through several levels seamlessly (Sarama & Clements, 2009, p. 21). With experience over time, the level of thinking may become robust, and children’s learning follows a predictable pattern (Barrett, Clements, & Sarama, 2017).

Another of the tenets of Hierarchic Interactionalism is the co-mutual development of concepts and procedures (tenet #5, Sarama & Clements, 2009, p. 22). We assume concepts can productively constrain procedures, and that concepts and procedures develop in mutual interaction (Baroody, Lai, & Mix, 2005; Greeno, Riley, & Gelman, 1984; Rittle-Johnson & Siegler, 1998). Thus, distinct from debates about which should be learned first (or focused on more intently), we posit that procedures can be learned meaningfully (connected with conceptual knowledge) or by rote—the latter we refer to as *mechanical procedures* (Hiebert & Wearne, 1993).

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