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When an argument is the content: Rational number comprehension through conversions across registers

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ABSTRACT

This article reframes previously identified misconceptions about repeating decimals by describing these misconceptions as limited understandings of how mathematics concepts are referenced. In particular, misconceptions about repeating decimals and their quotient of integer representations are recast as limited understandings of mathematics as a discipline that derives its content from representational systems and the denotations they provided. Under this framework, arguments (e.g., proofs) that convert repeating decimals to their quotient of integer representations provide content for "rational number," which is represented in multiple ways, each offering distinct opportunities for mathematical activity. The notion of an argument as content is illustrated as arguments providing access to a concept. One Grade 8 student's struggle with understanding rational number is used to illustrate this framework and its implications for teaching and learning.

1. Introduction

Prior research demonstrates that it is common for students and teachers to view repeating decimals as distinct from their quotient of integer representations (e.g., Burroughs & Yopp, 2010; Yopp, Burroughs, & Lindaman, 2011; Dubinsky, Arnon, & Weller, 2013; Dubinsky, Weller, McDonald, & Brown, 2005; Ely, 2010). Such views often remain intact after students and teachers are presented arguments showing that a repeating decimal is equal to a quotient of integer. Ultimately, these students view these distinct representations of the same real number as representations of two distinct numbers, or they view repeating decimals as never-ending processes that approximate numbers.

Progress has been made in helping learners view repeating decimals as mathematical objects equal to their quotient of integer representations (e.g., Dubinsky et al., 2013; Sinclair, Liljedahl, & Zazkis, 2006). However, the meta-level question of whether students understand that repeating decimals and well-defined quotients of integers are merely representations of the same concept, distinct from the concept itself, has not been addressed sufficiently. In particular, teaching and learning approaches that help students view "rational numbers"[1]; as a concept defined and represented in multiple ways for different mathematical purposes have not been explicitly explored in the mathematics education literature.

Previous researchers such as Yopp et al. (2011) used conversions from repeating decimal representations to quotient of integer representations as arguments to convince students and teachers that a repeating decimal is equal to its quotient of integer representation or to produce cognitive disequilibrium within students or teachers who did not view 0.999… and 1 as equal (Burroughs & Yopp, 2010; Yopp et al., 2011). The latter use of these arguments has helped researchers explore repeating decimal concepts more

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[1] I use quotations around "rational number" at times to distinguish between the concept of rational number and the concept's objectifications.

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deeply. However, these studies did not go so far as to explicitly describe the conversion arguments as providing content for the concept of rational number.

In this article, this viewpoint of conversions as providing content is addressed. Conversions across representations of "rational number" are recast as arguments that provide content for rational number that can lead to "rational number" comprehension.

From this vantage point, misconceptions about repeating decimals identified in previous literature are reframed as *limited understandings* of how mathematical concepts are accessed and referenced. In particular, student misconceptions are reframed within the literature on representation systems and the role of registers in providing content for a concept and its objectifications. This perspective situates *content* as arising from registers, and the denotations that registers provide, and as arising from the transformations within and between registers. Specifically, the problem of students' limited comprehension of "rational number" is reframed in terms of registers and reference systems, as motivated by Duval (2006).

After developing this idea in a theoretical framework, I demonstrate the framework by analyzing one Grade 8 student's conceptions of rational number and its various representations. I conclude by discussing implications for developing rational number comprehension through a framework for "*arguments as content through conversions across registers*."

I illustrate the framework by addressing the following research questions in the context of Grade 8 learning activities targeting "rational number" comprehension:

RQ1. What misconceptions does a Grade 8 student express about repeating decimals and their relationships to other representations of "rational number," and how similar are these misconceptions to those identified among undergraduates and in-service teachers?

RQ2. How can misconceptions about the relationships among different representations of rational number be viewed as limited understandings of mathematics as a collection of registers used to access abstract notions?

RQ3. How can conversions across rational number registers be viewed as arguments that provide content for "rational number?"

2. Literature on difficulties with "rational number" comprehension

Questions about individuals' understanding of the relationship between a repeating decimal and its quotient of integer representations have been addressed by exploring student understanding of the equality between $0.\overline{9}$ (henceforth represented as 0.999…) and 1. These studies have largely been confined to students in undergraduate courses such as calculus, real analysis, and mathematics for pre-service elementary teachers. One exception is Yopp et al. (2011), who explored the issue with Grade 5 teachers.

Students and some teachers are generally in a great deal of conflict over the relationship between repeating decimals and other number representations of rational number. This is particularly true when students and teachers are presented arguments showing that 0.999… and 1 are equal (e.g., Yopp et al., 2011; Ely, 2010). Tall and Schwarzenberger (1978), though not the first to conduct studies in this area, found that the majority of students in a first-year university course thought that 0.999… was less than 1, and this finding is fairly consistent across subsequent studies. Edwards and Ward (2004) found that some students think that 0.999… is distinct from 1 because, in their minds, 0.999… cannot be obtained as a result of long division. Such students view long division with integers such as 1/3 as never-ending, never-completed processes. More generally, some college students view "infinite repeating decimals as large but finite strings of digits, have difficulties believing that two different decimals can represent the same number, or think of infinite repeating decimals as approximations of their corresponding fractional representations" (Dubinsky et al., 2013, p. 235).

Students who view 0.999… as a number distinct from 1 often position 0.999… on a number line just to the left of 1 (Yopp et al., 2011). These students envision tiny or infinitesimal distances between 0.999… and 1. Difficulties with limits in calculus courses may stem from such views (Sierpinska, 1987). Yet, Ely (2010) pointed out that in some situations, students' beliefs in infinitesimal gaps between repeating decimals and their corresponding integer or quotient of integer representations can be developed into non-standard real number conceptions that are consistent and just as powerful as standard real number system conceptions.

Instructional interventions seem to help students change their views about the equality between fractions and their corresponding decimal representations (e.g., Dubinsky et al., 2013; Sinclair et al., 2006). Pre-service teachers can develop stronger, more stable beliefs that 0.999… = 1 after engaging in interventions that transition views of repeating decimals as actions or process into views of repeating decimals as objects that can be operated upon (Dubinsky et al., 2013; Weller, Arnon, & Dubinsky, 2009; Weller, Arnon, & Dubinsky, 2011). This transition appears to involve an intermediate stage where students shift their view of 0.999… as nines occurring as time passes to a view of 0.999… as a totality where all the nines occur at once (Dubinsky et al., 2013).

3. Theoretical framework

3.1. Learning "rational number" in the US common core

Rational number learning is a topic of interest to an international audience because rational number is central to mathematics that involves real number systems. Yet, each nation may have distinct standards and progressions for learning about rational numbers. The current study was performed in the United States, where Common Core State Standards (CCSS-M) suggest a progression for learning about rational number. Different nations may have different expectations for students. To communicate the context in which the study was performed, CCSS-M's suggestions are reviewed.