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An analysis of the form and content of quadrilateral definitions composed by novice pre-service teachers☆

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ABSTRACT

Mathematical definitions are an important mathematical construct which has been noted as a challenging topic for both teachers and students. This study provides an analysis of the form and content of a set of 308 definitions of quadrilateral types provided by 44 novice pre-service elementary teachers (NPSTs) who had not yet studied geometry or definitions on the college level. Analysis of definition structure including necessary, sufficient, and minimal conditions, as well as hierarchical and partitional structure, provides insight into what NPSTs may think about the form of mathematical definitions in general. Analysis of the properties used in definitions of each shape type and the frequency of those properties reveals the shape types with which NPSTs are most and least familiar. These results are presented alongside possible applications and implications for instruction.

1. Introduction

The research presented in this article uses literature-based characteristics of definitions as a means of categorizing definitions provided by novice pre-service teachers (NPSTs). An analysis of the form and content of these definitions of various kinds of quadrilaterals provides insights into NPSTs' conceptions of quadrilaterals and mathematical definitions in general. The majority of previous research done in the areas of how pre-service teachers (PSTs) think about quadrilaterals and their definitions has been conducted during or after the topics were studied in teacher preparation programs or only involved a few shape types (Thanheiser, Browning, Edson, Kastberg, & Lo, 2013; Zazkis & Leikin, 2008). The work presented in this article was conducted earlier in their professional development and involves a wide variety of shape types to provide a more comprehensive overview of the kinds of knowledge individuals bring into teacher preparation programs. I refer to participants in this study as "novice pre-service teachers" (NPSTs), meaning pre-service teachers who are in the early stages of their professional development and have not yet addressed the mathematical topic of study in college courses. NPSTs should not be confused with "pre-service teachers" (PSTs), the term often used to describe individuals who have completed a larger portion of their formative studies, or "novice teachers," the term often used to describe teachers during their first few years in the field. The present study provides insights into the kinds of knowledge and understanding NPSTs bring into teacher preparation programs, whereas similar data collected from PSTs who have completed a significant portion of their coursework and student teaching or novice teachers in their first few years of practice would provide insights into the kinds of knowledge and understandings individuals hold when leaving teacher preparation programs. The implications of these early findings are relevant for at least two reasons. First, they provide a delayed assessment of the effects of K-12 education on this population, which could have implications for the teaching of mathematics in the elementary, middle, and high

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school levels. Second, they provide useful information for mathematics teacher educators (MTE's) who wish to develop more productive strategies for supporting the development of NPSTs' thinking about quadrilaterals and mathematical definitions.

2. Pre-Service teachers and definitions of quadrilaterals

The way a NPST defines quadrilateral types can be influenced by both her understanding of mathematical definitions in general and her understanding of the concept being defined (Leikin & Winicki-Landman, 2001; Zazkis & Leikin, 2008). Studies of PSTs' understandings of mathematical topics have often found that "PSTs exhibit misconceptions identified in the school mathematics research literature and associated with emergent understandings" (Thanheiser, et al., 2013, p. 13). The following sections contain a synopsis of literature focusing first on PSTs' ways of thinking about definitions, and second on PST's ways of thinking about quadrilaterals.

2.1. Pre-service teachers' understanding of definitions

Mathematical definitions play an important role in the study of practically every area of mathematics (de Villiers, Govender, & Patterson, 2009; Usiskin & Griffin, 2008; Vinner, 1991). The study of definitions is a rich context that highlights logical relationships between mathematical statements, didactical sequences of learning, mathematical connections, and mathematical communications (Leikin & Winicki-Landman 2001). Knowing the form, content, and function of mathematical definitions is a vital communication tool for both teachers and students in K-12 mathematics (National Council of Teachers of Mathematics, 2000; National Governors' Association Center for Best Practices & Council of Chief State School Officers, 2010; Sinclair, Pimm, & Skelin, 2012). However, the concept of definition presents many challenges to teachers at various stages of their careers (Chesler, 2012; Levenson, 2012; Leikin & Winicki-Landman, 2001; Linchevsky, Vinner, & Karsenty, 1992). Students, mathematicians, and mathematics educators value many diverse and sometimes conflicting aspects of definitions such as: accuracy, mathematical elegance, minimal conditions, usefulness, clarity, and didactic practicality (Leikin & Winicki-Landman, 2000; Winicki-Landman & Leikin, 2000; Zaslavsky & Shir, 2005). These differing goals, combined with the fact that there are many logically-equivalent ways to define a concept, make mathematical definitions a challenging and complex mathematical construct (Leikin & Winicki-Landman, 2000; Usiskin & Griffin, 2008).

Researchers have found that although it can be challenging for students and teachers to effectively apply mathematical definitions (Chesler, 2012; Fujita, 2012; Levenson, 2012; Zaslavsky & Shir, 2005), PSTs think deeply about the concept of definitions and the work of defining concepts (Zazkis & Leikin, 2008). Teachers hold a wide variety of conceptions and values related to mathematical definitions (Leikin & Winicki-Landman, 2001). Some are similar to the variety of conceptions found among twelfth grade students (Fujita, 2012; Zaslavsky & Shir, 2005). Others are related specifically to the context of teaching, such as the appropriateness of including prior concepts as part of a definition, the virtues of a concise minimal definition versus a more-detailed and accessible one, and the benefits of rigorous definitions compared to definitions, which might seem pedagogically more appropriate (Zazkis & Leikin, 2008).

PSTs focus primarily on definitions as names (de Villiers et al., 2009) and less on their role as a part of a deductive system. This implication is further supported by the observation that a definition alone was not sufficient support to help PSTs successfully complete tasks that require unpacking and applying a mathematical concept (Cunningham & Roberts, 2010). This suggests that NPSTs may be able to name or recognize definitions, but may not yet fully understand the meaning behind them or their mathematical origins. PSTs often think of definitions in geometry as being based on some physical representation of a concept, when in reality both definitions and physical representations are ways of communicating about some general and abstract geometrical concepts (Kuzniak & Rauscher, 2005). A more thorough understanding of definition may help PSTs to conquer challenges similar to those of K-12 students, such as reliance on prototypical examples when working on solving problems involving hierarchical definitions (Fujita, 2012).

The variety of ideas about the structure of a mathematical definition and the challenges present when using mathematical definitions have moved researchers to suggest that the topic of definitions needs to be explicitly addressed as part of teacher preparation (Leikin & Winicki-Landman, 2001; Zazkis & Leikin, 2008). The act of writing mathematical definitions has been used as a means of helping teachers to unpack the form and purpose of mathematical definitions (Leikin & Winicki-Landman, 2001). In this article, I explore NPSTs' understanding of definition via their written definitions, specifically focusing on a definition's primary role of naming and identifying mathematical objects as examples and non-examples of a concept depending on whether or not they satisfy that definition.

2.2. Pre-Service teachers' thinking about quadrilaterals

Many students do not receive high-cognitive-demand geometry tasks during elementary or middle school (Battista, 2007) and often resort to memorization when faced with more complex tasks in high school (Burger & Shaughnessy, 1986). Unless they engaged with high-cognitive-demand geometry tasks in middle or high school, individuals' conceptions may not have changed much compared to those they developed in elementary school (Burger & Shaughnessy, 1986). These kinds of conceptions include: identifying shape types based on recognition rather than properties (Burger & Shaughnessy, 1986), using imprecise language when describing geometric attributes (Pegg & Davey, 1998), making assessments based on appearance of figures alone (Pegg & Davey, 1998), and attending to irrelevant non-geometric attributes of shapes such as size and orientation (Clements & Sarama, 2000; Monaghan, 2000). These ways of thinking could cause an individual to compose a definition that describes a typical example of a shape type rather than a more general description which includes all members of the class (Gutierrez & Jaime, 1998). Most textbooks include definitions that reflect a hierarchical classification of quadrilaterals, a way of thinking about the relationship between different kinds of quadrilaterals in which some shapes types are considered subsets of other shape types (Usiskin & Griffin, 2008). Yet both teachers and