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How mathematicians assign points to student proofs

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ABSTRACT

In this paper, we present an exploratory study on the important but under-researched area in undergraduate mathematics education: How do mathematics professors assign points to the proofs that their students submit? We interviewed nine mathematicians while they assigned points to three student-generated proofs from a transition-to-proof course. We observed that (i) One proof that contained a generic sub-proof was evaluated as correct by all nine participants and was given full credit by six participants, (ii) there were ten instances in which a mathematician did not assign full credit to a proof that she evaluated as correct, (iii) there was substantial variation in the points assigned to one proof, and (iv) mathematicians assigned points based not primarily on the correctness of the written artifact that they were given, but rather based on their models of students' understanding. We discuss the importance of these observations and how they can inform future research.

1. Introduction

In the United States, many mathematics majors are required to take a transition-to-proof course prior to taking proof-oriented courses such as real analysis and abstract algebra. In a typical transition-to-proof course, the expectation is that students will spend considerable time developing and mastering the mechanics of proof writing. The purpose of this paper is to focus on one aspect of the teaching of a transition-to-proof course: how mathematicians assess the proofs that their students submit for credit. There is a growing body of research on how proof is introduced to university students (e.g., Alcock, 2010; Hemmi, 2006; Nardi, 2008; Moore, 1994; Weber, 2004, 2012) and how students understand proof and related notions after completing a transition-to-proof course (e.g., Hawthorne & Rasmussen, 2015; Moore, 1994; Weber, 2010). There is little research on how proofs are assessed in these courses (Moore, 2016). Grading is an important part of pedagogical practice (e.g., Iannone & Simpson, 2011; Mejia-Ramos et al., 2012; Resnick & Resnick, 1992; Van de Watering, Gijbels, Dochy, & Van der Rijt, 2008); the grades that students receive on the proofs that they submit shape students' beliefs about what their professors value, what type of product is acceptable as a mathematical contribution, and how they should engage in proof writing. Consequently, understanding how proofs are graded is useful for understanding both the pedagogy of mathematicians and students' beliefs about proof.

In an exploratory study, Moore (2016) found that the four mathematics professors that he interviewed deemed their grading, including both the marks they assigned and the commentary they provided, an essential part of their teaching. In

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this paper, we present another exploratory study, delving into more depth on several of the issues that Moore looked at while focusing on mathematicians' perceptions of omissions in student-generated proofs.

2. Related literature

2.1. Proof in university mathematics

Proof is widely considered to be a cornerstone of mathematical practice. Consequently, an important aim of enculturating university mathematics students into the discipline of mathematics is engaging them in the activities of reading and writing proofs. A comprehensive review of the proving literature and a detailed discussion on the epistemology of proof is beyond the scope of this paper. However, we briefly review three key findings: First, most mathematics students at the university level struggle to write proofs (e.g., Iannone & Inglis, 2010; Weber, 2001; Weber & Alcock, 2004), even though proof writing is a skill emphasized in many advanced mathematics courses. Second, university mathematics students typically perform poorly when they are asked to distinguish valid proofs from invalid arguments (e.g., Inglis & Alcock, 2012; Selden & Selden, 2003; Weber, 2010), suggesting that students may submit invalid justifications because they cannot distinguish invalid justifications from invalid proofs. Third, some researches have suggested that many students are utterly perplexed about the enterprise of proof (e.g., Mamona-Downs & Downs, 2005). A recent comprehensive review of this literature is given in (in press) Stylianides, Stylianides, & Weber (2017).

In mathematics education, there has been an extended debate on what is, or should, constitute a mathematical proof (e.g., Balacheff, 2008; Cirillo et al., 2015). In this paper, we take an agnostic stance on this question and focus instead on what *mathematicians* consider to be a proof, at least in the context of a transition-to-proof course. Hence, by "proof", we simply mean a written artifact that a mathematician would evaluate to be a proof.

At least in North America, a pivotal experience in students' enculturation of proof occurs in the context of a transition-to-proof course. Mathematics majors typically take such a course in their sophomore or junior year, prior to taking theoretical proof-oriented courses such as abstract algebra and real analysis. In a transition-to-proof course, students are expected to learn both how to write proofs and about the nature of proof itself through a variety of activities, including having students practice applying a variety of proof techniques (e.g., proof by induction, proof by contradiction) across different mathematical contexts (e.g., Alcock, 2010; Moore, 1994).

2.2. Summative assessment in advanced mathematics

Summative assessments play an important role in the teaching of advanced mathematics. The assessments given in mathematics courses provide students with a clear indication of what mathematics professors value and exert an influence on the mathematics that students learn (Iannone & Simpson, 2011; Mejia-Ramos et al., 2012; Resnick & Resnick, 1992; Van de Watering et al., 2008). Iannone and Simpson (2011) claimed that research on assessment in collegiate mathematics courses is sparse. The existing research focuses primarily on assessment in calculus (e.g., Boesen, Lithner, & Palm, 2010; Bergqvist, 2007; Lithner, 2006) and the types of questions that students are asked to complete, rather than how students are assessed and how marks are assigned in upper-level proof-oriented courses.

In proof-oriented courses, some researchers have claimed that most assessments are largely comprised of proving tasks. For instance, Weber (2001) argued that students' ability to construct proofs is "typically the only means of assessing students' performance" (p. 101). Raman (2004) analyzed the exercises related to continuity in a real analysis textbook and found that most consisted of requiring the student to establish that a function was continuous or deduce some conclusion from the hypothesis that a function was continuous. Annie Selden (personal communication) analyzed the exercises in a typical real analysis textbook and found that over 80% were proving tasks.

2.3. Mathematicians' grading

While research on the types of assessment items that students are given in their advanced mathematics courses is emerging, there is little research on how mathematicians assign points to students' proofs in these courses. Here we summarize the main findings from Moore's (2016) and Lew and Mejia-Ramos' (2015) exploratory studies on this topic. Moore (2016) asked four mathematics professors to assign points to seven authentic student proofs with the aim of investigating the consistency, or lack thereof, in the marks that professors assigned. The main findings from Moore's study were that there was substantial variation in the scores they assigned to some proofs. Further, this variation that Moore observed was usually not due to one mathematician overlooking a flaw in the proof that another mathematician identified. Rather, the mathematicians simply disagreed how many points (if any) should be deducted from the same purported flaw that they both noticed. Moore proposed the following account for these disparities of grading: The mathematicians were using the written artifacts that students submitted to estimate how well students understood the proofs of the statements. The professors all remarked that grading was an important part of their pedagogical practice. Moore (2016) called for qualitative research to better understand the various criteria that professors use to grade proofs. The current study answers this call.

Lew and Mejia-Ramos (2015) presented eight mathematicians with proof fragments that were written in awkward unconventional language and they were told these proof fragments were from whole proofs that students submitted for

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