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Developing a contextualization of students' mathematical problem solving

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ABSTRACT

This paper investigates how students contextualize mathematical problem solving, not the actual problems. When students attempt to solve problems, what contexts (situational, cultural, or conceptual) do they evoke to describe their experiences with problem solving? The Common Core State Standards for Mathematical Practice emphasize contextualizing and decontextualizing problems, but what does this mean in practice? Middle and high school students were asked to attempt ability-appropriate problems during semi-structured interviews in this qualitative study. Situational contexts were analyzed using representation analysis (symbolic and nonsymbolic) while cultural contexts were analyzed using linguistic analysis (metaphors). The synergy of these two analyses developed a coherent and consistent conceptual contextualization for mathematical problem solving. Secondary students conceptualized problems as containers with the given information within the problem and solutions outside the problem. Thus students' representations are a means to travel from within the problem to outside of the problem.

1. Introduction

Reasoning abstractly, making sense of problems, and persevering during problem solving are critical elements of the Standards for Mathematical Practice (National Governors Association, Council of Chief State School Officers, 2010). Moreover, problem solving is central to mathematics and instruction should give students daily experiences with it (Kilpatrick, Swafford, & Findell, 2001). Despite this agreed upon significance, problem solving is difficult to define experientially (Stanic & Kilpatrick, 1989). In the last 60 years, mathematics educators have perceived mathematical problem solving as a heuristic process (Polya, 1945), a logic-based program (Newell & Simon, 1972), a means of inductive and deductive discovery (Lakatos, 1976), a framework for goal-oriented decision making (Schoenfeld, 1985, 2011), methodologies with multiple variables (Kilpatrick, 2004), a standard (NCTM, 1989), and a model-eliciting activity (Lesh & Zawojewski, 2007). Each contextualization of mathematical problem solving affects one's perception of what defines its purpose (Schoenfeld, 1992). For example, perceiving problem solving as discovery is epistemologically and pedagogically different from perceiving problem solving as a process (Silver, 1985). This study focused on the participants who had the most to gain or lose from their perception of problem solving, the students. This study asks:

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How do students' contextualize mathematical problem solving?

Students' contextualization of mathematical problem solving is difficult to organize, analyze, and conceptualize (Nilsson, 2009). Nilsson and Ryve (2010) offer two significant factors that can aid researchers in understanding contextualization: coherence and consistency (see Framework section). Using these two factors as criteria, this study draws upon interviews with three high-school students and three middle-school students. Students were asked to solve mathematics problems that were challenging, but ability-appropriate. Two researchers used a bricolage design (Cobb, 2007) focusing on specific aspects of students' responses. Researcher1 analyzed students' contextualization of mathematical problem solving via the students' language focusing on metaphors. Researcher2 analyzed students' contextualization of mathematical problem solving through their use of written symbolic and nonsymbolic mathematical representations. This collaboration was unique because it allowed identical data to be triangulated via distinct vantage points.

To clarify the interpretation of our findings, we will first discuss the overarching theoretical framework, participants, and procedures shared by both researchers. Second, we will discuss the framework, method, analysis, and results of linguistic metaphors students used to solve mathematics problems. Third, we will discuss the framework, method, analysis, and results of students' mathematical representations employed during problem solving. Finally, this study will synthesize both sets of results to identify how students contextualize mathematical problem solving.

2. Framework

2.1. Problem solving

As a framework, we define a problem as a developmentally appropriate challenge for which the participant has a goal but the means for achieving it are not immediately apparent (Polya, 1945; Schoenfeld, 2011). Often when solving problems, the existence of a solution is uncertain because the means to attain a solution is unknown (Lesh & Zawojewski, 2007; Pólya, 2004; Schoenfeld, 2011). Problem solving requires making sense of the problem situation and the means necessary for making decisions, which directs an individual's understanding (Schoenfeld, 2011). These three components for a problem are shared across the numerous characterizations of problem solving. However, more recent characterizations of problem solving include distinct constructivist elements that involve connecting the situational context within the problem to experiences shared by the problem solver. More specifically, research on students' problem solving indicates that prior experiences and knowledge, beliefs and dispositions, and culture play a huge role in how individuals approach problem solving (Lesh & Zawojewski, 2007; Schoenfeld, 2011). Students experiencing rich problem-solving instruction have better problem-solving outcomes than peers in exercise-laden learning environments (Bostic, 2011; Lesh & Zawojewski, 2007). Thus, prior experiences influence students' problem-solving performance and approaches and in turn, scholarly interpretations of students' problem solving.

2.2. Coherence and consistency

Interpreting perceptions of students attempting to solve mathematical problems is the purpose and crucial theory within this study. When interpreting data from student interviews, it is vital to understand students' representations of how they would solve a problem. Constructivist epistemology challenges the means in which one interprets representations because the interviews only demonstrate *re-presentations* (von Glasersfeld, 1991). von Glasersfeld (1987) clarifies differences between re-presentation and representation within radical constructivism by stating:

> "Because perceiving, from a constructivist point of view, is always an active making, rather than a passive receiving, the similarity of a picture and what it depicts does not reside in the two objects but in the activities of the Experiencer who perceives them." (p. 217)

von Glasersfeld is emphasizing the distinction between a picture and the genuine object as perceived by the experiencer. To the experiencer, the picture is a symbolic representation, while the genuine article is the iconic representation (von Glasersfeld, 1987). The term re-presentation refers to the understanding that if one constructed their own knowledge, even the iconic representation "is an artifact and a deliberate reconstruction of another experiential item" (von Glasersfeld, 1987, p. 217). Consequentially, all iconic and symbolic representations of an individual are mental re-presentations of the individual's experiences.

The distinction between re-presentation and representation is significant because this study is not attempting to justify one re-presentation as correct for problem solving. If this were even possible, such justification would require prior knowledge of all students' cognitive schema. Instead, this study strives to attain coherence amongst an individual's multiple iconic and symbolic representations (von Glasersfeld, 1991). The notion of coherence is directly linked to the methodology of this study and is to be measured by its consistency within an individual as well as across participants. This study directly asks if mental re-presentations are *close enough* using Nilsson and Ryve's work (2010) with coherence and consistency.

Similar to Nilsson and Ryve (2010), we define objects as *coherent* if significant traits of the objects similarly coalesce for a specific purpose. Additionally, we define objects as *consistent* if the significant traits reoccur frequently. Epistemologically,