Learning and Individual Differences 64 (2018) 104–112

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Arithmetic practice that includes relational words promotes understanding of symbolic equations☆

Dana L. Chesney[a,*], Nicole M. McNeil[b,**], Lori A. Petersen[c], April E. Dunwiddie[b]

[a] Department of Psychology, St. John's University, New York, United States
[b] Department of Psychology, University of Notre Dame, United States
[c] Department of Psychology, Purdue University Northwest, United States

A R T I C L E I N F O

Keywords:
Mathematics learning
Arithmetic practice
Conceptual understanding
Early algebra
Symbolic understanding

A B S T R A C T

We tested whether using relational words to highlight the relational nature of equality during arithmetic practice can improve what children learn from that practice. Children were randomly assigned to one of four addition practice conditions: (a) relational words: equality symbols were sometimes replaced with relational words [e.g., "is the same amount as"], (b) traditional symbols only: equality was only symbolized with the equal sign or equal bar, (c) operational words: equality symbols were sometimes replaced with operational words [e.g., "adds up to"], or (d) no extra practice. As hypothesized, the relational words group showed evidence of a more relational understanding of symbolic equations at posttest: they were more likely than the other groups to encode the "=" when asked to reconstruct pre-algebra problems, to show transfer from addition practice to subtraction, and to use advanced solving strategies. Results suggest children can benefit from minor arithmetic practice modifications that highlight the relational aspects of equality.

1. Introduction

Many elementary school children (ages 7–11) have a poor understanding of arithmetic concepts, despite being proficient at calculating answers to simple addition and subtraction problems. For example, many do not understand that adding and then subtracting the same number from a given set results in no change to the starting value of the set (e.g., $3 + 5 - 5 = 3$; Baroody, Torbeyns, & Verschaffel, 2009). Moreover, they may not understand that knowing $3 + 4 = 7$ can help them answer $4 + 3 =$ _ or $7 - 4 =$ _ (Baroody et al., 2009; Robinson & Dubé, 2009; Siegler & Stern, 1998), nor that the two sides of an arithmetic problem are interchangeable (e.g., $3 + 4 = 7$ can be written $7 = 3 + 4$, Kieran, 1981). One hypothesized source of individual differences in children's understanding of these arithmetic concepts is children's ability to stop and engage in relational thinking before calculating when presented with symbolic arithmetic problems (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). *Relational thinking* refers to "looking at expressions and equations in their entirety, noticing number relations among and within these expressions and equations" (pg. 260). Individual differences in relational thinking predict mathematics achievement (McNeil, Hornburg, Devlin, Carrazza, & McKeever, 2017). In the present study, we tested this hypothesis experimentally by activating children's relational thinking during arithmetic practice and observing how it affects what children learn from that practice.

Researchers used to think that children's poor understanding of arithmetic concepts resulted from something children lack compared with adults, such as domain general knowledge or working memory capacity (Case, 1978; as cited in Kieran, 1980; Piaget & Szeminska, 1941/1995). However, a growing body of research indicates that children's misconceptions are due to the overly narrow way in which arithmetic facts are presented and studied in elementary school (e.g., Baroody & Ginsburg, 1983; McNeil et al., 2012; McNeil & Alibali, 2005b; Seo & Ginsburg, 2003; Sherman & Bisanz, 2009). Basic arithmetic facts are one of the first topics of study in children's formal mathematics education. Children are often taught the simplest facts, such as "$1 + 1 = 2$" and "$2 + 2 = 4$", before they have mastered the count list or base-10

☆ Author note: Dana L. Chesney was at the University of Notre Dame and is now at St. John's University, New York. Lori A. Petersen was at the University of Notre Dame and is now at Purdue University Northwest. This research was supported by Grant R305B070297 from the U.S. Department of Education, Institute of Education Sciences to McNeil. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education. Thanks to Martha Alibali and Heather Brletic-Shipley for their input. Thanks also to Emily Fyfe for help with coding and tutoring, and Caroline Hornburg for her help. This research would not have been possible without the support of many administrators, teachers, parents, and students in the greater South Bend area.

* Correspondence to: D. L. Chesney, Department of Psychology, Marillac Hall, Room 409, St. John's University, Queens Campus, 8000 Utopia Pkwy, Jamaica, NY 11439, United States.
** Correspondence to: N. M. McNeil, Department of Psychology, University of Notre Dame, 118 Haggar Hall, Notre Dame, IN 46556, United States.
E-mail addresses: chesneyd@stjohns.edu (D.L. Chesney), nmcneil@nd.edu (N.M. McNeil).

https://doi.org/10.1016/j.lindif.2018.04.013
Received 18 May 2017; Received in revised form 23 April 2018; Accepted 28 April 2018
1041-6080/

notation (Klein, 2005). Although there are benefits from gaining proficiency with arithmetic facts, there are also limitations. The rote drilling of arithmetic facts in the traditional way does not convey the conceptual foundations of arithmetic (see Brownell, 1935; Schneider & Stern, 2009).

Arithmetic is typically taught with little reference to the equal sign as an indicator of equality. Problems are nearly always presented in a unidirectional "operations on left side" format, with all operators on the left and the answer on the right (McNeil et al., 2006; Powell, 2012; Seo & Ginsburg, 2003). Moreover, equality is typically expressed only with the equal sign in horizontally written problems or the equal bar in vertically written problems, rather than described in a fashion that more explicitly conveys the equivalence relation between the two sides of an equation. These narrow experiences might lead children to construct overly narrow interpretations of the equal sign, which are not easily overcome. According to a *change-resistance account* (e.g., McNeil & Alibali, 2005b), acquisition and entrenchment of these incorrect interpretations serve as a barrier to constructing a formal, relational understanding of symbolic equations (Knuth, Stephens, McNeil, & Alibali, 2006; McNeil et al., 2006; Steinberg, Sleeman, & Ktorza, 1990). Instead of coming to understand that the "=" in equations symbolizes that items share the same value, children may become entrenched in the idea of the equal sign, and equations more generally, as commands to perform arithmetic operations or calculate results. This entrenched *operational thinking* carries with it the idea that "=" signals a unidirectional process. While children may understand that "2 + 2 = 4", they will not agree that "4 = 2 + 2", claiming that these statements are incorrect or nonsensical (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Kieran, 1980).

These negative consequences of traditional practice with arithmetic are unacceptable, but eliminating arithmetic practice and relying on calculators is not a viable alternative. Early emphasis on memorizing basic facts stems from the notion that children need to have a firm grounding in basic arithmetic before they can develop an understanding of higher-level mathematics (see Brownell, 1935). It has been suggested that mastery of basic facts "frees up" cognitive resources for more advanced mathematical thinking (Klein, 2005; National Research Council, 2001). Individuals who rely mostly on retrieval strategies, rather than calculation strategies, for single-digit arithmetic problems perform better on standardized tests (Price, Mazzocco, & Ansari, 2013), and further, proficiency with arithmetic facts aids pattern recognition and inductive reasoning (Haverty, 1999).

Fortunately, the acquisition of such operational ways of thinking is not inevitable. Children educated in China and Korea, countries whose education practices are less narrow and promote relational thinking, do not typically demonstrate operational interpretations of the equal sign and can solve mathematical equivalence problems correctly (Capraro et al., 2010; Li, Ding, Capraro, & Capraro, 2008). We posit—and indeed research has shown—that small modifications to the traditional approach to arithmetic practice that discourage the entrenchment of and reliance on operational thinking can be beneficial to conceptual understanding of arithmetic. For example, research has shown benefits of practicing arithmetic problems in nontraditional formats such as _ = 2 + 2 (Baroody & Ginsburg, 1983; McNeil, Fyfe, Petersen, Dunwiddie, & Brletic-Shipley, 2011; Seo & Ginsburg, 2003; Weaver, 1973), grouping math facts with equivalent values during practice, like 6 + 4 = 10 and 5 + 5 = 10 (McNeil et al., 2012), and seeing the equal sign in non-arithmetic contexts, such as 4 = 4 or 3 feet = 1 yard (McNeil, 2008; see also Li et al., 2008). What these methods have in common is that they highlight the relation between the quantities on both sides of the equal sign. This notion of the equal sign as a *relational* symbol is foundational for understanding algebra (Falkner, Levi, & Carpenter, 1999; Fyfe, Matthews, Amsel, McEldoon, & McNeil, 2018; Knuth et al., 2006).

The present study evaluated a potential modification to traditional arithmetic practice that may activate relational thinking—using relational words to highlight the relational nature of the equal sign. Past research has shown that children gain more from a brief lesson on an arithmetic problem with operations on both sides of the equal sign (e.g., 3 + 4 + 5 = 3 + _) when the lesson provides a relational description of the equal sign (e.g., "the amount before it needs to equal the amount after it") than when it provides step-by-step instructions on how to solve problems (Rittle-Johnson & Alibali, 1999). Similar interventions using relational words in place of the equal sign in adults learning novel arithmetic facts have been shown to benefit transfer between complementary arithmetic facts (Chesney & McNeil, 2014). It follows that arithmetic practice including relational words (e.g., "is equal to," "is the same amount as") may help children make more conceptual gains from arithmetic practice.

To test this prediction, we randomly assigned children to one of four conditions. In the *relational words* experimental condition, children practiced simple addition problems that included relational words (e.g., 3 + 4 "is the same amount as" _). In the *traditional symbols* active control condition, children practiced the same addition problems with traditional equality symbols only (i.e., equal sign and equal bar). In the *operational words* active control condition, children practiced the same addition problems with operational words that children typically use to describe the equal sign (e.g., "adds up to"). There was also a *no practice* non-intervention control condition in which children did not receive any facts practice beyond what they typically receive at school or at home. We hypothesized that children in the relational words groups would gain more conceptually from the arithmetic facts practice than children in the control groups.

Our study closely followed the procedures of McNeil and colleagues (McNeil et al., 2011; McNeil et al., 2012): a posttest-only experiment with random assignment at the individual level. The use of a randomized intervention-posttest design, rather than a randomized pretest-intervention-posttest design was crucial to this study's validity. Mere exposure to mathematical equivalence problems has been shown to improve children's performance (Alibali & Meredith, 2009). Moreover, effect sizes of interventions designed to improve understanding of mathematical equivalence are magnified by the use of a pretest (McNeil et al., 2012). As such, it is more conservative to assess our intervention without a pretest, as the use of a pretest would have undermined the study's ability to determine the benefit of the intervention itself, rather than in combination with the pre-exposure effects.

In essence, the goal of the relational words intervention was to see if experimentally inducing relational thinking during arithmetic facts practice would foment conceptual change. While conceptual change in non-core domains is notoriously difficult (Gelman, 2009), prior studies have achieved some improvement in relational understanding of symbolic equations with conceptually-based interventions of just a few weeks (McNeil et al., 2011; McNeil et al., 2012). We did not expect the intervention to yield mastery of the underlying arithmetic concepts, but rather sought to detect the beginnings of this change. The posttest included three tasks: a measure designed to assess children's formal understanding of mathematical equivalence, which is the concept that the two sides of an equation are equal and interchangeable (McNeil et al., 2011; McNeil et al., 2012); a standardized arithmetic test (the Iowa Test of Basic Skills, ITBS); and a computerized simple addition task. Better performance on the measure of understanding of mathematical equivalence, in particular, indicates a more relational understanding of equations and, thus, provides evidence that the intervention improved conceptual understanding of arithmetic. The ITBS and the simple addition task were primarily included to confirm that the relational words intervention did not, as a side effect, decrease the effectiveness of the addition facts practice in bringing children to mastery of addition fact retrieval (an important skill, as we note above). However, some aspects of performance on these tasks can also indicate improvements in conceptual understanding. Relational understanding should facilitate understanding of the inherently relational concept of the complement principle, that if A + B = C, C − B = A (Bryant, Christie, & Rendu,