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Kindergartners' base-10 knowledge predicts arithmetic accuracy concurrently and longitudinally

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ABSTRACT

Children's early knowledge of the base-10 structure of multi-digit numbers has been hypothesized to play a critical role in subsequent learning of mathematics, in particular arithmetic operations. The present study investigated the relation between base-10/place value understanding and arithmetic accuracy in early elementary school. Children were assessed in kindergarten ($N = 90$) and then a subgroup of participants was assessed again two years later in second grade ($N = 21$). Mediation analyses indicated that, in kindergarten, base-10 knowledge had a direct effect on arithmetic accuracy as well as an indirect effect through the use of a decomposition strategy. Furthermore, kindergarten base-10 knowledge had a direct effect on arithmetic accuracy in second grade and an indirect effect through second grade place-value notation understanding. Implications for understanding early mathematical development are discussed.

An understanding of the base-10 system is posited to be a critical aspect of early mathematics knowledge (Geary, 2006; Miura, 1987; NCTM, 2000; National Research Council, 2001). More specifically, it is widely believed that base-10 knowledge is necessary for accurate computation of multi-digit arithmetic problems (Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001). Errors in carrying and borrowing in written addition problems, for instance, have been attributed to a lack of understanding of base-10 and place value (Brown & Burton, 1978; Fuson, 1990; Hiebert, 1997; Ross, 1986; Varelas & Becker, 1997). Further, base-10 knowledge is related to the use of decomposition strategies, which are most efficient for solving problems with sums above 10 (Laski, Ermakova, & Vasilyeva, 2014). To date, however, discussion of the relation between base-10 understanding and arithmetic problem solving has been primarily theoretical (e.g., Fuson & Briars, 1990; Geary, Hoard, Nugent, & Bailey, 2013). In the present study, mediation analyses were used to simultaneously examine the relations among kindergartners' base-10 knowledge, addition strategies, and addition accuracy and to test whether kindergartners' early base-10 knowledge predicts accuracy on more complex multi-digit problems in second grade.

1. Base-10 and place-value notation

It takes several years for children to develop an understanding of the base-10 system and place-value notation (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson, 1986, 1988, 1992; Fuson & Briars, 1990; Ginsburg, 1989; Varelas & Becker, 1997). Before formal schooling, most children think of numbers larger than ten as collections of units rather than as groups of tens and units (Mix, Prather, Smith, & Stockton, 2014). Children's understanding of the base-10 numeric structure is typically assessed with a block-task (e.g., Miura, Okamoto, Kim, Steere, & Fayol, 1993) in which children are asked to "show" two-digit numbers using blocks that include small cubes representing single units and bars that represent ten units combined together. If children think of numbers as collections of single units, they will show a number, such as 32, using 32 individual unit cubes. If, however, children understand the base-10 structure of numbers, they are more likely to show a number, such as 32, using three ten-bars and two individual units. Between kindergarten and second grade, children increasingly use both tens- and single units to represent two-digit numbers (Miura, 1987; Miura et al., 1993; Saxton & Towse, 1998).

Thinking of multi-digit numbers as groups of tens and units should translate into later place-value notation knowledge because it lays the foundation for understanding that the numeric magnitudes represented by each digit vary based on the digit's position in a number. Suggestive of this relation, one of the most common misconceptions about place-value notation—concatenation—reflects a lack of understanding of the base-10 structure of multi-digit numbers. A child who makes a concatenation error focuses on the face-value of digits as opposed to a digit's value as a multiple of ten based on its location within a multi-digit

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number (Cobb & Wheatley, 1988; Fuson & Briars, 1990; Miura et al., 1993; Price, 1997; Ross, 1989; Varelas & Becker, 1997). For example, when asked to describe how the numeral 24 relates to 24 sticks, children making face value errors may state that the 2 in 24 represents 2 sticks and the 4 represents 4 sticks, rather than 20 and 4 (Price, 1997; Ross, 1989). Children's understanding of place-value notation and its application to arithmetic increases during elementary school. In a cross-sectional study, Varelas and Becker (1997) found that the percentage of children who traded correctly and gave the correct digit for the 10s place on a written arithmetic task increased from 56% to 77% to 98% between second and fourth grades. In the present study, the relation between early base-10 knowledge and later place-value notation understanding was examined empirically using a longitudinal design.

2. Base-10, addition strategies, and accuracy

To be successful on more complex problem solving in math, children must first learn to accurately and efficiently solve simple arithmetic problems in early elementary school (Cowan et al., 2011; Jordan, Kaplan, Olah, & Locuniak, 2006). Children can arrive at solutions to addition problems through various types of strategies. When asked to solve problems without paper and pencil, children typically use one of three types of addition strategies: counting, decomposition, and retrieval (Geary, Bow-Thomas, Liu, & Siegler, 1996a, 1996b; Geary, Fan, & Bow-Thomas, 1992; Shrager & Siegler, 1998). Counting strategies involve enumerating both of the addends or counting-up from one of the addends. The retrieval strategy involves recalling the solution to a problem as a number-fact stored in memory, rather than active computation. Decomposition involves transforming the original problem into two or more simpler problems, and often begins with solving for ten first (e.g., base-10 decomposition: solving $6 + 5$ by adding 6 and 4 to get to 10, and then 1 more).

A decomposition strategy is useful for solving arithmetic problems, particularly when the problems involve sums above ten and/or double-digit addends (Ashcraft & Stazyk, 1981; Torbeyns, Verschaffel, & Ghesquiere, 2004). Children and adults who frequently use decomposition to solve arithmetic problems tend to have higher math performance and overall math achievement scores than those who depend on counting strategies (Carr & Alexeev, 2011; Carr, Steiner, Kyser, & Biddlecomb, 2008; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). A recent study found that the frequency with which first graders' use a decomposition strategy predicted their accuracy on complex addition problems and mediated cross-national differences in accuracy on these complex arithmetic problems (Vasilyeva, Laski, & Shen, 2015).

Kindergartners who use decomposition tend to have a better understanding of the base-10 structure of the number system than those who do not (Laski et al., 2014). This relation makes sense theoretically. Consider, for example, addition problems with single-digit addends that require carryover into the tens place (e.g., $7 + 5$). Having a base-10 understanding of two-digit numbers (e.g., $12 = 10 + 2$) may facilitate the use of decomposition in solving this problem: $7 + 5 = (7 + 3) + 2 = 12$. Similarly, better understanding of base-10 structure may facilitate use of decomposition strategy in solving problems with multi-digit addends, with or without carryover (e.g., $23 + 14$). In order to use a decomposition strategy that involves adding tens, then ones, then combining the results, a child must know which digits represent the tens and be able to increment by tens rather than ones. Therefore, it is not surprising that understanding of base-10 structure predicts the use of decomposition. Identifying base-10 knowledge as a predictor of decomposition strategy use by children is noteworthy because this strategy has been shown to lead to higher arithmetic accuracy (e.g., Geary et al., 2004). Importantly, however, the relation among base-10 knowledge, use of decomposition strategy, and arithmetic accuracy has not been investigated in the context of a single study within the same group of children. Thus, no direct evidence for the theoretical relation between these three aspects of mathematics knowledge exists. The present study empirically tested this relation.

3. The present study

The present study had two primary goals. The first goal was to simultaneously examine the relations among kindergartners' base-10 knowledge, addition strategies, and addition accuracy. Based on the analysis above, we expected that the extent to which children represented double-digit numbers as tens and ones, rather than as a collection of units, would predict their accuracy on addition problems and that this relation would be mediated by the frequency with which they used a decomposition strategy.

The second goal of the study was to examine longitudinally the relation between early base-10 knowledge and later mathematics performance. A subgroup of kindergartners was followed to second grade and asked to answer place value notation questions as well a set of more complex arithmetic problems. We expected that kindergartners' base-10 knowledge would predict their understanding of place-value notation two years later. Further, we expected that early base-10 knowledge would influence later arithmetic accuracy through place-value notation understanding because place-value notation understanding is essential for solving complex arithmetic problems that involve multi-digit addends and carrying and borrowing.

4. Method

4.1. Participants

The present study included a group of kindergartners ($N = 90$; *Mean age* = 6;1 years, $SD = 0.34$). In addition to testing the full sample in kindergarten, we were able to test a subset of the sample two years later when children were in second grade ($N = 21$; *Mean age* = 8;3 years, $SD = 0.42$). Analyses indicated no differences on kindergarten

Table 1
Coding scheme for addition strategies.

Category	Definition	Behavioral/verbal cues	Example
Counting	Enumerating each unit in one or both of the addends	Child verbally counted by one or exhibited counting through behavioral cues during problem solving or reported enumerating addend(s) when describing the solution.	6 + 5… "7, 8, 9, 10, 11" "I counted 5 from 6."
Decomposition	Transforming the original problem into simpler problems, using base-10 properties or previously memorized number facts	Child reported several steps involving breaking the original addends into smaller numbers. This could be observed during problem solving or during the child's explanation.	6 + 5… "6 + 4 = 10, 10 + 1 = 11"
Retrieval[a]	Recalling a required number fact from memory	Child reported the answer within 3 s with no overt evidence of counting or decomposition stated that he/she "just knows the answer"	6 + 5… 11, "I just know it."
Other	None of the above	Child reported guessing or not knowing, or reported a strategy that was unclear and could not be clarified by further prompting	"I don't know."

[a] Retrieval is only used on single-digit problems.