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Developmental trajectories of calculation and word problem solving from third to fifth grade

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ABSTRACT

The current study focuses on the developmental trajectories of calculation and word problem solving skills using person-oriented approach. The sample included 882 Estonian children form 29 schools. Calculation, word problem solving, verbal skills, and planning were assessed in Grades 3, 4 and 5. Latent class analysis and latent transition analysis were used. The number of latent classes was explored each year and the developmental trajectories were found with latent transition analysis. In each year, there were groups of children with overall high or overall low skill levels, as well as groups with similar math scores but diverse results in verbal and planning skill levels. The results indicate that to better support math skill development, children could benefit from working on tasks that enhance specifically planning and verbal skills; e.g., learning the steps of planning, solving the tasks of word guessing.

1. Introduction

In recent years, there have been discussions about the mismatch of methods and theories in the developmental research (Bergman & Andersson, 2010; Sterba & Bauer, 2010), which is also noticeable in the area of math development (e.g., Fuchs et al., 2010). Variable-oriented methods use variables in the statistical analyses to construct or support a theory, and this type of statistical model has linear relationships. With variable-oriented methods, it is assumed that the same model is applicable to the large portion of the sample. However, it can be argued from a developmental perspective that non-linear relationships exist, thus, variable-oriented methods may entail weaknesses in that research area (Bergman & Andersson, 2010). While theoretical approaches emphasize the interactive nature of cognitive abilities, preskills, and environmental factors in academic (including math) development, previous studies have mostly used variable-oriented methods that enable to examine mainly linear relations between variables. Person-oriented approach, in contrast, searches for groups of individuals with different developmental pathways that may end up with the same result (Bergman, Magnusson, & El-Khouri, 2003).

Employing the person-oriented approach may help to explain controversial findings shown by variable-oriented methods. For example, Jordan, Hanich, and Kaplan (2003) showed that children with math difficulties had similar levels of problem solving skills as did the children with reading difficulties; however, these groups of children might have made different errors in solving the tasks. Students with math difficulties may have problems with rapid fact retrieval and their procedural or conceptual delays can be associated with addition and subtraction errors (Jordan & Montani, 1997). Students with reading difficulties may have weaknesses in word recognition and comprehension (Jordan & Montani, 1997) which can lead to other types of errors (e.g., understanding the phrase "two times less" as an indication to subtract two units from the whole). These differences may include not only general mathematical skills, but also calculation and word problem solving skills separately. Improvement in calculation skills may not improve word problem solving skills and vice versa (Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014). Problem solving skills have been shown to be more stable and develop independently from basic calculation skills from the first through the fourth grade (Vukovic & Siegel, 2010). Moreover, studies have generally shown the important role of verbal skills and planning in the development of math skills (Kikas, Peets, Palu, & Afanasjev, 2009; Mazzocco & Kover, 2007; Naglieri & Gottling, 1997; Tolar et al., 2012; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). Because their influence on problem solving skills tends to be higher than on calculation skills due to the more complicated nature of problem solving (Cowan & Powell, 2014; Fuchs et al., 2010), different developmental pathways may be expected for children with low problem solving skills. So far, these possible developmental pathways have not been studied. Finding different developmental pathways can contribute to developing prevention and intervention strategies considering math skills.

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Thus, the current study aims to examine the development of calculation, word problem solving, verbal, and planning skills with person-oriented methods to add new perspectives to the existing literature. Verbal skills and planning were included due to their important but different roles in solving calculation and word problems (Geary, 2011; Jordan et al., 2003; Mazzocco & Kover, 2007; Naglieri & Gottling, 1997; Tolar et al., 2012). Moreover, their general importance in academic development has been emphasized by several theories (e.g., functional system theory, see Luria, 1973; Planning, Attention, Simultaneous, Successive theory, see Das & Naglieri, 1995). We assessed children at three time points during the transition from lower elementary to upper elementary grades (Grades 3–5), when math problems become more difficult and demanding. The study was carried out in Estonia, where students have shown high math skill levels in international comparative studies, for example, the Programme for International Student Assessment (OECD, 2013), but the general level of math achievement in National Achievement Tests is much higher in the third grade compared with the sixth grade (Üleriigiliste tasemetööde tulemused, 2013), thus we aim to get further information how math skills could be improved throughout the school years taking into consideration verbal skills and planning.

1.1. Calculation and word problem solving skills

During the first school years math learning concentrates largely on calculation skills. Calculation requires the processing of operational symbols or words, retrieval of basic arithmetical facts, and procedural skills (McCloskey, Caramazza, & Basili, 1985). At the end of lower elementary grades, children mainly retrieve arithmetic facts from memory and use calculation techniques to produce the answers (Siegler, 1996). Skills in arithmetic fact retrieval play an important role in computational estimation (Booth & Siegler, 2006) and solving complex multi-digit calculations (Andersson, 2008). Basic calculation tasks are also related to conceptual knowledge (Cowan et al., 2011) and children who understand how numbers can be decomposed and combined use more advanced models of problem solving (Canobi, Reeve, & Pattison, 2003). Moreover, problem solving skills predict pre-algebraic knowledge (Fuchs et al., 2012).

Word problems that are used in primary school can be classified as change, combine, compare, and equalize problems (Jordan & Hanich, 2000). Change problems have unknown result, unknown change or unknown start meaning that the initial quantity is increased or decreased by specific action. Combine problems are the problems where two quantities that do not change must be considered in combination. In compare problems the difference between two quantities must be found. Equalize problems involve two separate quantities, one of which needs to be equalised with the other quantity (Carpenter & Moser, 1984; Jordan & Hanich, 2000; Riley & Greeno, 1988). Word problems can be considered more complex if they need more than one step to reach the solution, have greater text-based demands, have more realistic presentation, and include integration across math domains (e.g., computation and graph reading) or nonessential details (Fuchs & Fuchs, 2002; Fuchs, Fuchs, & Prentice, 2004). Thus, word problem solving includes understanding the connections between known and unknown quantities (Fuchs et al., 2012), translating each sentence into numerical or mathematical information (Fuchs et al., 2003), and requires combining procedural and conceptual knowledge. Good problem solvers use advanced counting procedures, understand part-whole relations in calculation tasks and use their knowledge flexibly (Canobi, 2004). Gray, Pinto, Pitta, and Tall (1999) suggest that in solving math problems, less successful children focus more on the specific aspects and do not see the generalizable sides of the problem, while high achievers focus more on the flexible aspects and have a wider conceptual view of the problem. However, routine word problems can also be solved without understanding the whole problem and only applying known procedures (Pólya, 1945); understanding the problem and using wider conceptual view of the problem is important in solving non-routine problems (Gray et al., 1999). Thus, children with lower mathematical skills may concentrate more on basic procedures rather than on general concepts.

Complex and multistep problems require decisions about the problem type and solving procedure (Fuchs et al., 2003); therefore, understanding the problem's representation and developing solution plans and strategies are important parts of problem solving (Andersson, 2008; Pólya, 1945; Wu & Adams, 2006). According to Mayer (1982a), students use qualitatively different solution strategies for word and calculation problems, but word problems do not always require more effort. He also suggests that students have specific schemas for every problem type, so difficulties may arise when they are given a problem for which they have no schema. Thus, he proposes that students should practice determining similar and different problem types (Mayer, 1982b).

1.2. The role of verbal skills and planning in solving calculation tasks and word problems

Verbal skills, such as integrating verbal information, comprehending words from given descriptions, and drawing conclusions about concepts, have been found to be related to both calculation and problem solving skills (Männamaa, Kikas, Peets, & Palu, 2011). In solving word problems it is necessary to read and extract all the information from the question to turn the problem into a mathematical model (Wu & Adams, 2006); thus, math word problem solving is related to reading fluency (Hart, Petrill, Thompson, & Plomin, 2009) and text comprehension (Vilenius-Tuohimaa et al., 2008). Difficulties in translating mathematical word problems into numerical or mathematical information can come from different sources. Text comprehension, knowledge about the meaning of symbols, signs, and words used in mathematical texts, and ability to use and organize prior knowledge play an important role in solving mathematical word problems. Some students may know the meaning of words and symbols used in texts, yet are not able to connect different parts of the texts (Duru & Koklu, 2011). Although reading problems and math difficulties are related and earlier math difficulties can even predict later reading problems (Lin et al., 2013), there are children who have math difficulties without any reading problems (Vukovic & Siegel, 2010). Thus, children with math difficulties should be looked at as a separate group from children with combined math and reading difficulties, because children with only math difficulties may have an advantage over children with combined math and reading difficulties (e.g., Hanich, Jordan, Kaplan, & Dick, 2001).

Planning is an executive function that involves programming, regulation, and verification of behaviour. Through planning students are able to solve problems, ask questions, and control their impulses (Luria, 1973). According to the PASS (i.e., Planning, Attention, Simultaneous, Successive) theory, planning is required when the individual must make decisions about how to solve a problem, execute an approach, activate attentional, simultaneous, and successive processes, monitor the effectiveness of the approach, and modify actions as needed (Das & Naglieri, 1995). Therefore mathematical problem solving and planning are tightly related and improving planning skills can also improve math skills (Naglieri & Gottling, 1997).

Compared with calculation tasks, solving word problems tends to require a wider range of domain-general abilities (Cowan & Powell, 2014; Fuchs et al., 2010). Calculation tasks are usually already set up for a solution, while students have to use the text to identify the missing information, construct the number sentence, and derive the calculation problem for finding the missing information for word problems (Fuchs et al., 2008). This could make word problems more complex and substantially different from calculation tasks. For example, Seethaler and Fuchs (2006) showed that computational estimation skills may develop independently from verbal skills. However, early arithmetic skills can be one of the competences that override the influence