Learning and Individual Differences 49 (2016) 297–304

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Children's multiplication and division shortcuts: Increasing shortcut use depends on how the shortcuts are evaluated☆

Katherine M. Robinson [a,*], Adam K. Dubé [b], Jacqueline-Ann Beatch [c]

[a] Department of Psychology, University of Regina, Canada
[b] Department of Educational and Counselling Psychology, McGill University, Canada
[c] Department of Psychology, University of Calgary, Canada

ARTICLE INFO

Article history:
Received 26 June 2015
Received in revised form 4 March 2016
Accepted 24 June 2016

Keywords:
Arithmetic
Inversion
Associativity
Conceptual knowledge
Multiplication
Division

ABSTRACT

The goals of this study were to investigate whether children's use of two arithmetic shortcuts based on the understanding of the operations of multiplication and division could be increased and to examine how children's evaluations of the shortcuts interacted with the brief task aimed at promoting shortcut use. Grade 6, 7, and 8 students solved two sets of inversion (e.g., $d \times e \div e$) and associativity (e.g., $d \times e \div f$) problems. Children with a good understanding of the relation between multiplication and division can use conceptually-based shortcuts to solve both types of problems. Students were also given a brief task demonstrating the inversion shortcut (the answer is the first number and no calculations are required) and the associativity shortcut (dividing first and then multiplying) and were asked to compare each shortcut to a left-to-right procedure (i.e., multiplying and then dividing). Half the participants were given the demonstration task between the problem sets and the other half after the sets. Inversion and associativity shortcut use increased by Grade 8 and improved across problem sets for both the Demonstration Middle and Demonstration Last groups. The demonstration task successfully promoted subsequent shortcut use but the participants who positively evaluated the shortcuts compared to the left-to-right procedure in the demonstration task had greater subsequent shortcut use. Conceptually-based shortcut use on multiplication and division problems was increased using a brief demonstration task but children's subsequent use of shortcuts depended on their evaluations of the shortcuts.

1. Introduction

Numerous studies from mathematics education and cognitive development have demonstrated the difficulties that children have with conceptual knowledge of arithmetic (Prather and Alibali, 2009). We define conceptual knowledge as the understanding of the underlying structure of arithmetic and that the understanding of this structure can be used as the basis for problem solving procedures (also see Crooks and Alibali, 2014). The focus of the current study is on children's understanding of the relationships between arithmetic operations which are particularly challenging for children (Gilmore and Papadatou-Pastou, 2009). These difficulties, especially with concepts relating to the operations of multiplication and division (Robinson, Ninowski, and Gray, 2006b), have rarely been investigated (Robinson and LeFevre, 2012) but highlight the need for research to improve conceptual understanding which is critical for the development of mathematical knowledge (Kilpatrick, Swafford, and Findell, 2001), and for later algebraic reasoning (Nunes et al., 2008).

The challenge of conducting research on children's conceptual knowledge is in assessing that knowledge (Bisanz and LeFevre, 1990) but two concepts provide important information about children's knowledge. The inversion concept, extensively researched on addition and subtraction problems, is the understanding that addition and subtraction and multiplication and division are pairs of operations that are inversely related to one another (Starkey and Gelman, 1982). When children understand that inverse relationship, they can apply their knowledge by using a procedure called the inversion shortcut on problems such as $3 + 24 - 24$ or $3 \times 24 \div 24$ (Bisanz and LeFevre, 1990). In these problems, adding and subtracting or multiplying and dividing by the same number results in the first number remaining unchanged. The inversion shortcut requires no calculations and is fast and accurate (Bisanz and LeFevre, 1990). Shortcut use is commonly used as an index of conceptual understanding (Crooks and Alibali, 2014; Gilmore and Papadatou-Pastou, 2009; Prather and Alibali, 2009) but may underestimate conceptual knowledge if children do not apply their knowledge during problem solving (Canobi, 2009).

☆ This research was supported, in part, by an NSERC discovery grant. Thank you to Jen Gibson, John Brand, and Anna Maslany for assistance with the collection and coding of the data. Many thanks to the children, teachers, and principals from the participating schools.

* Corresponding author at: Campion College at the University of Regina, 3737 Wascana Parkway, Regina, Saskatchewan S4S 0A2, Canada.
E-mail address: katherine.robinson@uregina.ca (K.M. Robinson).

http://dx.doi.org/10.1016/j.lindif.2016.06.014
1041-6080/

The associativity concept is the understanding that addition and subtraction and multiplication and division are pairs of operations which are associatively related to one another, and therefore each operation within a pair can be solved in any order (Klein and Bisanz, 2000). When children understand the associative relationship, they can apply their knowledge by using a procedure called the associativity shortcut to simplify the problem solving process on problems such as $3 + 24 - 20$ or $3 \times 24 \div 6$ by solving $24 - 20$ or $24 \div 6$ first (Robinson and Dubé, 2009a). Solving the subtraction or division component first simplifies problem solving as children are now dealing with smaller numbers (e.g., $3 \times 24 \div 6$ becomes 3×4) and results in reductions in solution times and fewer retrieval or calculation errors (Robinson and LeFevre, 2012).

Students' weak understanding of the relationships between multiplication and division is "unacceptable and indicates a substantive gap in the mathematics curricula that must be addressed (National Mathematics Advisory Panel, 2008, pp. 4–39)." Little research has been conducted on the multiplication and division inversion and associativity concepts (Robinson and LeFevre, 2012) let alone on how to strengthen children's understanding of these concepts. What is known is that both children and adults consistently use fewer conceptually-based shortcuts on multiplicative than additive versions of inversion and associative problems (Robinson and Ninowski, 2003; Robinson et al., 2006b), use the associativity shortcut less than the inversion shortcut (Robinson et al., 2006b), that large individual differences exist in both additive and multiplicative shortcut use, and that additive shortcut use reaches adult levels in middle childhood but multiplicative shortcut use does not reach adult levels until adolescence (Dubé, 2014). Given these difficulties, it is particularly important to increase understanding of multiplicative concepts.

No research has been conducted on how to increase children's understanding of the multiplication and division concepts of inversion and associativity unlike the addition and subtraction inversion and associativity concepts. For addition and subtraction inversion problems, repeated exposure (Siegler and Stern, 1998), training with concrete objects (Lai, Baroody, and Johnson, 2008), and visual and oral shortcut demonstrations (Nunes, Bryant, Hallett, Bell, and Evans, 2009) have all increased shortcut use.

An alternative, and much briefer, approach was used by Robinson and Dubé (2012, 2013) and was based on the premise that the ability to recognize the validity of a conceptually-based procedure or shortcut implies knowledge of the concept and that the ability to also recognize its superiority to a familiar and well-learned left-to-right procedure indicates even stronger understanding (Bisanz, Watchorn, Piatt, and Sherman, 2009; Canobi, Reeve, and Pattison, 1998; Prather and Alibali, 2009). Robinson and Dubé (2009a) assessed children's understanding of addition and subtraction inversion and associativity concepts using two tasks. First, Grade 2 through 4 children's conceptual understanding of inversion and associativity was assessed via shortcut use during problem solving. Second, children were then shown inversion and associativity problems and given a verbal demonstration for each problem type of how one fictitious child used a shortcut, and a verbal demonstration of how another fictitious child used a left-to-right procedure. Participants were asked to compare the pair of procedures on each problem type and decide which approach (the shortcut or the left-to-right procedure) was better. Most children preferred the inversion shortcut but only marginally preferred the associativity shortcut.

Robinson and Dubé (2012, 2013), also with addition and subtraction, used the same problem solving and demonstration tasks but had half the Grade 2 through 5 participants complete the demonstration task in the middle of the problem solving task. The demonstration task increased later shortcut use, particularly for the associativity shortcut. Importantly, however, children's evaluation of the demonstrated procedures mitigated shortcut use. Children who, during the demonstration task, recognized the superiority of a shortcut versus a left-to-right procedure had much greater subsequent shortcut use than children who did not. The demonstration task shows promise for being an effective tool for quickly promoting shortcut use and understanding but its effectiveness depends on children's evaluations of the shortcuts.

No study has explicitly attempted to promote the inversion or associativity concepts for multiplication and division three-term problems. Robinson and Dubé (2009b) investigated repeated exposure to multiplication and division inversion problems using the same design as Siegler and Stern (1998). They found that over a third of Grade 6 students who were repeatedly exposed to multiplication and division inversion problems failed to discover the multiplication and division inversion shortcut. This suggests that the multiplication and division version of the inversion concept is more complex and that repeated exposure alone is insufficient to promote children's conceptual understanding.

The demonstration task used by Robinson and Dubé (2012, 2013) on addition and subtraction problems is promising as it can be easily adapted to multiplication and division problems. Robinson and Dubé (2009c) used the demonstration task to assess conceptual understanding of inversion for multiplication and division but only after problem solving. Unlike addition and subtraction, participants in Grades 6 through 8 who compared the inversion shortcut to a left-to-right procedure only marginally preferred the inversion shortcut and inversion shortcut use was lower than found on addition and subtraction problems (see also Robinson et al., 2006b). Taken together, the findings from these studies are disconcerting because children near the end of middle school need to be prepared to learn algebra and they should be fluent with multiplication and division by Grade 5 (National Mathematics Advisory Panel, 2008). To be successful in algebra, one important factor is the understanding of the relationships amongst operations (Kilpatrick et al., 2001). The results of work on the multiplication and division concepts suggest that children's understanding of these relationships is weak and therefore it is critical to find a way to increase this understanding.

In previous research, children's attitudes towards the inversion and associativity shortcuts have differed across individuals (Robinson and Dubé, 2009c, 2012, 2013). While some children considered the shortcuts to be clever approaches to problem solving, others considered the shortcuts to be a form of cheating (i.e., skipping procedural steps). That children have strong feelings about how mathematics should be performed is not surprising even though it has rarely been investigated in the domains of either procedural or conceptual knowledge of arithmetic. Other research has shown that, as schooling proceeds, children develop diverging beliefs, attitudes, motivations, and anxieties towards mathematics (Beilock, 2008; Lepper, Corpus, and Iyengan, 2005; Martin, Anderson, Bobis, Vellar, and Way, 2012; McLeod, 1993; Wolters, 2004). Thus, we expected that the demonstration task would help increase conceptually-based shortcut use on multiplication and division problems as it did on addition and subtraction problems but that it would be most effective for students who positively evaluated the shortcuts (Robinson and Dubé, 2012, 2013). However, given that previous work has shown that children use the inversion and associativity shortcuts much less frequently on multiplication and division problems than on addition and subtraction problems and are also more skeptical about the shortcuts themselves (Robinson and Dubé, 2009c; Robinson et al., 2006b), it is possible that the demonstration task will not be sufficient to promote shortcut use.

The goals of the current study were to establish for the first time, (a) whether understanding of the inverse and associative relationships between multiplication and division can be successfully promoted using a task previously successful for promoting the same relationships between addition and subtraction and (b) whether the success of this task would be mitigated by children's evaluations of the conceptually-based shortcuts.