Learning and Individual Differences 35 (2014) 103–112

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

# Mathematics growth trajectories in first grade: Cumulative vs. compensatory patterns and the role of number sense

Martin Salaschek, Nina Zeuch, Elmar Souvignier *

*Department of Psychology, University of Münster, Fliednerstraße 21, D-48149 Münster, Germany*

ARTICLE INFO

*Article history:*
Received 14 November 2013
Received in revised form 16 April 2014
Accepted 28 June 2014

*Keywords:*
Growth trajectories
Mathematics
Number sense
Progress monitoring

ABSTRACT

We examined mathematics growth trajectories in first grade for overall achievement and three separate competences (Basic Precursors, Advanced Precursors, Computation). 153 German students computed seven web-based progress monitoring tests during the school year. Latent class growth analysis (LCGA) provided evidence for mainly cumulative patterns of performance development: In all competences, we found groups of initially high-performing students with the highest end scores and groups of initially low-performing students with little or no growth. In contrast to this general pattern, compensatory trajectories with groups of initially lower-performing students and steep growth were found. For precursor competences, these catch-up groups did not have increased odds of belonging to low-end outcome groups in higher competences. The findings demonstrate the added value of repeatedly assessing both precursor and grade-level skills for identifying students who follow favorable or unfavorable learning trajectories.



## 1. Introduction

During the last decade, there has been growing interest in the developmental dynamics of children's math performance during the first years of education. Several studies have used longitudinal data to model learning trajectories of different groups of students, providing remarkable new insight into the stability of students' long-term mathematical performance (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Bodovski & Farkas, 2007; Geary, Hoard, Nugent, & Bailey, 2012; Jordan, Kaplan, Oláh, & Locuniak, 2006; Morgan, Farkas, & Wu, 2009). All of these sophisticated studies found that students with low math performance at the beginning of the study were likely to show substantially less learning growth than their peers with higher initial performance. However, most of the studies used measures of overall mathematical performance to categorize students into trajectory groups over several years. Less is known about the shorter-term heterogeneity in the development of early math skills and about the interplay of different competences. The aim of the present study thus was to examine different growth trajectories of students' overall math performance and of hierarchic competences in grade 1.

### 1.1. Early mathematics skills

Basic mathematical competences, such as knowledge about quantities and numbers, counting abilities, or basic arithmetic abilities, start to develop before school entry and broaden during the first years of formal education. These competences, commonly referred to as *precursor* or *number sense* competences, have not been consistently defined (for an overview, see Berch, 2005), but there is widespread agreement on their importance for students' further mathematical development (e.g., Chard et al., 2005; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Krajewski & Schneider, 2009; Locuniak & Jordan, 2008; Missall, Mercer, Martínez, & Casebeer, 2012).

Number sense sub-skills can be categorized with regard to the sequence of their development. First, children learn several basic skills, such as discriminating between quantities and identifying number words as quantities. When children understand the number (word) sequence as incrementing quantities, they acquire a "precise quantity-number linkage" (Krajewski & Schneider, 2009, p. 515). We refer to these skills as *Basic Precursors*. Basic Precursors make way for more advanced skills, e.g., recognizing number patterns and identifying a number on a number line (Dehaene, Piazza, Pinel, & Cohen, 2003; Siegler & Booth, 2004). Such tasks require the integration of several basic quantity-related skills and should mainly be developed before school entrance, as they are highly predictive of longer-term math achievement (Krajewski & Schneider, 2009). We refer to these skills as *Advanced Precursors*. Finally, developing *Computation* competence,

* Corresponding author. Tel.: +49 251 83 34308.
*E-mail addresses:* salaschek@uni-muenster.de (M. Salaschek), nina.zeuch@uni-muenster.de (N. Zeuch), elmar.souvignier@uni-muenster.de (E. Souvignier).

http://dx.doi.org/10.1016/j.lindif.2014.06.009
1041-6080/

such as addition and subtraction skills, is the main curricular goal in math instruction in the first school years. Fundamental understanding of composing and decomposing numbers into other numbers forms the basis for the most important curricular goals of the elementary school grades and is expected to be developed by the end of first grade even if initial competences are low.

As a limitation of sequential development models, skill developments are not strongly sequential. E.g., most of the mentioned precursor competences can be acquired without knowledge of Arabic notation, and knowledge of Arabic numbers can be acquired independently from other precursor skills. Still, children need to develop connections of Arabic numbers, number words, and the meaning of quantities, as depicted in Dehaene's triple-code model (Dehaene, 1992, 2001, 2011; Dehaene & Cohen, 1995). Moreover, as Krajewski and Schneider (2009) note, children can reach different competence levels for smaller and larger numbers, making it hard to accurately determine a child's competence level.

Research about children's development of mathematical competences should therefore take diverse precursor abilities on different developmental levels into account. Little is known, though, about typical sequences of number sense development because children learn fast, and longitudinal studies assessing several competences in short intervals are rare. Knowledge about varying growth trajectories in different math competences may help researchers and educators better understand the preconditions of a favorable or unfavorable long-term development, which can in turn be used to improve systematic early interventions.

### 1.2. Growth trajectories in mathematics

Growth trajectories in general-education mathematics have been more intensely studied using longitudinal multivariate methods for about a decade (Aunola et al., 2004; Bodovski & Farkas, 2007; Geary et al., 2012; Jordan et al., 2006; Morgan et al., 2009). The main aim of these studies is to describe how students systematically differ in their skills development over time and what characterizes different trajectory groups. This knowledge may help with early identification of students at risk of developing persistent difficulties in math and with designing trajectory-specific interventions.

#### 1.2.1. Growth trajectories of overall math performance

For the description of growth trajectories among students, cumulative or compensatory developmental patterns are discussed in the literature. Cumulative patterns, i.e., increasing differences in performance and variance between students over time, have been described by a number of studies in reading and mathematics (e.g., Bast & Reitsma, 1997; Bodovski & Farkas, 2007; Kempe, Eriksson-Gustavsson, & Samuelsson, 2011; Leppänen, Niemi, Aunola, & Nurmi, 2004; Williamson, Appelbaum, & Epanchin, 1991). This effect, with biblical reference, is also called *Matthew effect* (Stanovich, 1986). In contrast, a compensatory effect describes a pattern where initially less skilled students, with the beginning of formal instruction, show higher growth rates than initially higher-skilled children. As a consequence, the achievement gap between students narrows over time. This effect to our knowledge has first been described by Aarnoutse and van Leeuwe (2000) and subsequently in other studies for reading (e.g., Parrila, Aunola, Leskinen, Nurmi, & Kirby, 2005; Phillips, Norris, Osmond, & Maynard, 2002), but evidence for mathematics is scarce. The mixed evidence may suggest that there are certain conditions (e.g., certain characteristics of schooling or certain patterns of precursor abilities) under which students follow either trajectory path.

The number of studies that use a person-oriented approach to identify distinct mathematics growth trajectories increases slowly. Aunola et al. (2004) followed 194 Finnish children from kindergarten to the end of grade 2. Using growth mixture modeling, two trajectory groups were obtained: A *high performers* class was defined by a high overall level of performance and a high and positive growth rate. A *low performers* class was characterized by lower overall performance and a lower (but also positive) growth rate. Therefore, the results of the study suggest a cumulative pattern of math performance. No distinct small group that performs well below average was found in this study, as would be suggested by studies that identify groups of 5–20% of students with specific or broad learning difficulties (see Geary, 2004 and Mazzocco, 2005 for discussions of prevalences). This may be due to the homogeneous characteristics of the study sample.

Jordan et al. (2006) and Jordan, Kaplan, Locuniak, and Ramineni (2007) assessed the performance of about 400 students from kindergarten to the middle of grade 1 in a more heterogeneous sample. Using growth mixture modeling, Jordan et al. (2007) found three classes of overall number sense performance that were named by their outcome level and slope characteristics: a *low/flat* group displayed low performance at the last time point and only small competence gains over time; a *middle/steep* class started only slightly higher than the first group but displayed significantly steeper growth; finally, a *high/flat* class started out in kindergarten scoring about twice as high as the first group and displayed growth levels in-between the other two groups (scoring highest at the end of the study). Although this pattern may suggest a compensatory effect for some children, there were ceiling effects for the highest-scoring class, and it is unknown whether this class would have shown higher growth if more complex tasks had been used. In their 2006 study, using only the four time points in kindergarten, Jordan and colleagues found three very similar groups. However, growth of the *high* group was slightly higher than growth of the *middle* group during this shorter time frame.

In contrast to these studies that use data-driven methods to obtain trajectory characteristics and group sizes, several studies used normative performance criteria to categorize students into trajectory groups. Morgan et al. (2009) analyzed performance level and growth from kindergarten to fifth grade for children showing different math difficulties (MD) patterns in kindergarten. With extensive data from the US-representative Early Childhood Longitudinal Study — Kindergarten Cohort study (ECLS-K), the lowest-performing 10% of students were categorized as having MD in fall and/or spring of kindergarten. Students who did not display MD in either fall or spring of kindergarten showed the highest performance level and growth, followed by students displaying MD in fall of kindergarten only, students displaying MD in spring of kindergarten only, and finally students displaying MD in both fall and spring of kindergarten.

Bodovski and Farkas (2007) also used ECLS-K data to analyze students' performance from kindergarten to third grade. The authors divided students into four equally-sized groups according to their kindergarten fall performance. This approach yielded in very different categorizations than the approach used by Morgan et al. (2009) because the no-MD group in the Morgan et al. study roughly comprised the proportion of students who were represented in the three higher quartiles in the Bodovski and Farkas study. Vice versa, all three MD groups of the Morgan et al. (2009) study were roughly represented in just the lowest quartile in the Bodovski and Farkas (2007) study. Nonetheless, major conclusions about growth trajectories were similar: the higher the initial performance level, the higher the growth — with the exception of the two highest quartiles, which did not differ in their growth.

In sum, all five studies found at least one low-performing group of students with low learning growth over time, and one or more groups with higher overall performance and higher learning growth. These results seem to indicate that students generally follow a cumulative pattern during early development in mathematics.

#### 1.2.2. Growth trajectories of discrete math competences

Growth trajectories of discrete skills were analyzed in detail by Geary et al. (2012), who followed 177 students from first through fifth grade and assessed several measures of math achievement, along with other competences. With respect to their mathematics achievement,