Learning and Individual Differences 23 (2013) 101–107

Contents lists available at SciVerse ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Children's additive concepts: Promoting understanding and the role of inhibition ☆

Katherine M. Robinson *, Adam K. Dubé

Department of Psychology, Campion College at the University of Regina, Canada

ARTICLE INFO

Article history:
Received 30 August 2011
Received in revised form 10 July 2012
Accepted 23 July 2012

Keywords:
Addition
Subtraction
Conceptual knowledge
Inhibition
Inversion
Associativity

ABSTRACT

This study investigated the promotion of children's understanding and acquisition of arithmetic concepts and the effects of inhibitory skills. Children in Grades 3, 4, and 5 solved two sets of three-term addition and subtraction problems (e.g., 3 + 24 − 24, 3 + 24 − 22) and completed an inhibition task. Half of the participants received a demonstration of conceptually-based shortcuts between problem sets. All participants increased their use of the inversion shortcut (stating that the answer to a problem of the form $a+b-b$ was the first number without any calculations) across a problem set but only the participants who received the demonstration increased their associativity shortcut use (subtracting first and then adding on a problem of the form $a+b-c$), particularly if they evaluated the shortcut as being better than a traditional left-to-right computational algorithm (adding then subtracting). Four clusters of participants using varying degrees of conceptually-based shortcuts were identified. Participants with weak inhibition skills were more likely to use a strategy that mixed conceptual knowledge and a computational algorithm suggesting that although they had conceptual knowledge, they had difficulties inhibiting a well-learned computational algorithm.

1. Introduction

Children's understanding of arithmetic concepts is a key component of mathematical development (National Mathematics Advisory Panel, 2008). Mastery of additive concepts such as the inversion and associativity concepts demonstrates an understanding of the relationship between addition and subtraction (Prather & Alibali, 2009). If children understand that addition and subtraction are inversely related to one another, a problem such as 6 + 28 − 28 can be quickly and accurately solved by stating the first number and not performing any calculations (Starkey & Gelman, 1982). If children understand that addition and subtraction are associatively related to one another, a problem such as 6 + 28 − 23 can be more quickly and accurately solved by subtracting then adding versus adding then subtracting (Robinson & Ninowski, 2003). Research on the inversion concept has outpaced the associativity concept, but questions remain about how to promote conceptual understanding and what individual factors are related to conceptual knowledge. The study's goals were to investigate whether (1) promotion of conceptually-based shortcuts via a brief demonstration task could be achieved and (2) inhibition accounts for the individual differences in conceptually-based shortcut use.

1.1. Promotion of conceptually-based shortcuts

Much research has focused on how early children's conceptual understanding begins (e.g., Sherman & Bisanz, 2007) but few studies have aimed to promote it. Siegler and Stern (1998) found that repeated exposure to addition and subtraction problems led to all Grade 2 participants discovering and using the inversion shortcut. Lai, Baroody, and Johnson (2008) and Nunes, Bryant, Hallett, Bell, and Evans (2009) found that training with concrete objects led to improvements in conceptual understanding of inversion for 4-, 5-, and 8-year-olds.

These studies suggest that exposure to more inversion problems can lead to increases in understanding of inversion but training is more beneficial albeit requiring time and effort. Little is known about promoting the associativity concept but it is a more difficult concept for both children and adults (Robinson & Dubé, 2009; Robinson & Ninowski, 2003). Robinson and Dubé (2009) gave children a 5-minute brief demonstration of the inversion and associativity shortcuts and asked them to compare each shortcut to a traditional left-to-right problem solving strategy (e.g., 4 + 27 − 27 or 4 + 27 − 25 would be solved by adding 4 + 27 and then subtracting the third number). Many participants commented on the cleverness of one or both shortcuts but were not given the opportunity to use the shortcuts during subsequent problem solving. These comments led to the hypothesis that this task could promote both inversion and associativity shortcut use. Comparing problem solving strategies on mathematics problems is an effective learning tool (Rittle-Johnson & Star, 2007, 2009). Comparison encourages children to evaluate, internalize, and generalize information and can lead to deeper conceptual understanding (Gentner, 2005). We hypothesized that the demonstration task would lead to gains in

☆ This research was supported in part by an NSERC discovery grant awarded to the first author. Thank you to Anna Maslany and Tenille Ulmer for their assistance with the collection and coding of the data. Many thanks to the children, teachers, and principals from the participating schools.

* Corresponding author at: Campion College — University of Regina, 3737 Wascana Parkway, Regina, SK, Canada S4S 0A2. Tel.: +1 306 359 1248; fax: +1 306 359 1200.
E-mail address: katherine.robinson@uregina.ca (K.M. Robinson).

1041-6080/$ – see front matter
http://dx.doi.org/10.1016/j.lindif.2012.07.016

shortcut use. Robinson and Dubé (2009) also found that some students considered the shortcuts to be cheating — suggesting that it is possible for children's evaluation of shortcuts to impact subsequent problem solving strategies. Children often critique information they are presented with before adopting that information (Gelman, 2009) but previous research has not examined children's evaluations of the information they are given during inversion concept training. We hypothesized that the demonstration task would lead to greater shortcut use but that it would be most effective for children who evaluated the shortcuts as being better than a left-to-right strategy. Thus, the first goal of this study was to examine the effects of a brief method for promoting conceptual knowledge of both inversion and associativity and determine whether children's evaluations of the shortcuts would impact later shortcut use.

1.2. Individual differences, inhibition, and use of conceptually-based shortcuts

There are large individual differences in the use of conceptually-based shortcuts but the factors relating to these differences are poorly understood. Robinson and Dubé (2009) identified three clusters of children based on their strategies for solving inversion and associativity problems. The dual concept cluster had good understanding of both concepts, the inversion concept cluster had good understanding of the inversion concept, and the no concept cluster had weak understanding of both concepts. Gilmore and Papadatou-Pastou (2009) identified three subgroups of school-aged children based on their use of the inversion shortcut and on other measures of mathematics abilities. One subgroup had good understanding of the inversion concept and calculation skills, the second had poor understanding and calculation skills, and the last had good understanding but poor calculation skills suggesting that mathematics ability does not relate to conceptual knowledge. Neither study found that age predicted cluster or subgroup membership. Likewise, Rasmussen, Ho, and Bisanz (2003) found little relationship between working memory and understanding of the inversion concept in school-aged children.

One promising factor may be inhibition. Siegler and Araya (2005) proposed that one of the mechanisms necessary to discover and implement new strategies is the need to interrupt a procedure during problem solving to implement a new one. For inversion, children would need to interrupt an ongoing procedure – the left-to-right strategy – and apply their conceptual knowledge of the inverse relationship between addition and subtraction. This mechanism is supported by the finding that children commonly use the negation strategy before the inversion shortcut (Siegler & Stern, 1998). Negation involves adding the first two numbers and then realizing that subtracting the third number negates the addition of the second number (Bisanz & LeFevre, 1990). Negation represents a left-to-right strategy that is interrupted part way through problem solving by applying the knowledge that addition and subtraction are inverse operations. Siegler and Araya's (2005) model could suggest that children need to inhibit their tendency to implement left-to-right problem solving procedures and instead use a conceptually-based procedure such as the inversion or the associativity shortcut. If children have the conceptual knowledge of the inverse and associative relationships between addition and subtraction, they should be able to implement the related shortcuts unless they are unable to inhibit the well-established left-to-right approach to problem solving. MacLeod (2007) stated: “Cognitive inhibition is the stopping or overriding of a mental process, in whole or in part, with or without intention” (p. 5).

Previous research has shown that children with lower mathematical ability are poorer inhibitors (Bull, Johnston, & Roy, 1999, Bull & Scerif, 2001; St. Clair-Thompson & Gathercole, 2006) while children who are better inhibitors are better calculators (Swanson, 2006) and better word and algebra problem solvers (Agostino, Johnson, & Pascual-Leone, 2010; Khng & Lee, 2009; Passolunghi & Siegel, 2004). No study has examined conceptual knowledge and inhibition. We hypothesized that children who use conceptually-based problem solving strategies, such as dual concept users, may have greater inhibitory ability than those who do not use them. Children who frequently use shortcuts should be good inhibitors as they are able to interrupt a left-to-right strategy. The second goal of this study was to examine the role of inhibition as a factor relating to individual differences in conceptually-based shortcut use on three-term addition and subtraction problems.

2. Method

2.1. Participants

Participants were 36 Grade 3 students (10 boys), 32 Grade 4 students (13 boys), and 36 Grade 5 students (19 boys) (mean ages of 8:3, 9:3 months, and 10:5 in years:months, respectively). They were from a large Canadian city, predominantly Caucasian, and from schools in middle SES neighborhoods. The study took place in the first half of the school year.

2.2. Materials and procedure

There were two sessions conducted in quiet locations in the schools. In the first individually-administered session, participants solved a pretest and posttest each composed of 16 three-term addition and subtraction problems, and a demonstration task. Half of the problems in each problem set were inversion problems of the form $a+b-b$ (e.g., $3+6-6$ and $7+23-23$) and half were associativity problems of the form $a+b-c$ (e.g., $3+6-4$ and $7+23-21$). No more than two problems of each type were presented consecutively. Problems were presented on a laptop using e-prime. Solution latencies, accuracy, and immediately retrospective verbal reports of problem solving strategy were collected for each problem (i.e., “How did you solve the problem?”). Participants had a maximum of 10 s to solve each problem. If they were unable to solve the problem within the time limit, their response was coded as a “cut-off” and inaccurate but they were still asked to report how they had tried to solve the problem.

Half of the participants in each grade were randomly placed into the demonstration group and given the demonstration task from Robinson and Dubé (2009, the assessment of procedures task) between the pre and posttests. The task took approximately 5 min. On both an inversion and an associativity problem, two problem solving strategies were demonstrated to the demonstration group. On the inversion problem, the inversion shortcut (i.e., when the same number is added and subtracted, the answer will be the first number) and a left-to-right strategy (i.e., add the first two numbers and then subtract the third number to get the answer) were demonstrated. On the associativity problem, the associativity shortcut (i.e., subtract the third number from the second and then add the first number) and the same left-to-right strategy were demonstrated. For each problem type, the demonstration group participants were asked if each solution method was a good way of solving the problem (yes or no) and then which of the two methods was better. Problem type order was counterbalanced as equally as possible as was the order of the demonstrated strategies for each problem type. The control group solved the pre and posttests consecutively with only a brief interruption to indicate that they had finished the first set and would now solve a second set of problems and then they completed the demonstration task.

In the second session, groups of one to four participants were given the Stop Signal task from Lee, Ng, and Ng (2009). The Stop Signal task is a common measure of inhibition in children (e.g., St. Clair-Thompson & Gathercole, 2006) and adults (e.g., Friedman & Miyake, 2004). Words (e.g., ball, deer) appeared on the laptop screen and participants had to categorize them as animal or non-animal for two blocks of 48 trials. In the third block, on 48 random trials out of 192, a computer-emitted tone was heard and participants were instructed not to categorize the word on that trial and wait until the next word appeared on the screen. They then categorized the words