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Multiple symbolic representations: The combination of formula and text supports problem solving in the mathematical field of propositional logic

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ABSTRACT

As multiple representations are common in math education, we examined different combinations of representations (text, formula, graphic) in the field of propositional logic. In two experiments, we investigated whether participants benefit from multiple representations, and whether the kind of representational combination affects performance. For the first experiment, 146 university students were divided into six groups: two single-representation groups (text or formula), three dual-representation groups (all possible pairs), and one triple-representation group. Results indicate that the combination of multiple symbolic representations (text + formula) was as helpful as combining analog and symbolic representations (text + graphic and text + formula + graphic): the participants provided with all such types of multiple representations outperformed all single representation groups. In a second experiment ($N = 19$, within-subjects) we compared the gaze behavior when working with multiple symbolic representations (text + formula) or combined symbolic and analog representations (text + graphic). The results demonstrate that text was the one representation that was attended most and can therefore be regarded as the reference representation in all useful combinations. Even though fewer gaze switches were observed between text and formulas than between text and graphics, performance (time on task and accuracy) did not differ. This research emphasizes the importance of various forms of multiple representations in mathematics learning and assessment, and sheds light on how different kinds of representations and their combinations are processed.

1. Introduction

Specific learning content can be provided to the learner by different representational forms such as text, pictures, or animations. Frequently, more than one representation of a concept is used to foster learning and problem solving. There is a vast amount of empirical evidence that such multiple external representations (MERs) can be beneficial for learning (e.g., Mayer, 2009). However, their supportive effect is not universal but depends, for example, on the learners' prior knowledge (e.g., Kalyuga, Chandler, & Sweller, 2000), and on the characteristics of the combined representations (e.g., Kalyuga, Chandler, & Sweller, 2004). Subsequent research in this area is particularly concerned with exploring the conditions under which MERs are most effective (Ainsworth, 2006).

Studies of multimedia learning usually focus on combining representations that are heterogeneous regarding their forms of representation: either in modality (visual or auditory) or in their type of code (symbolic or analog). Current theories of multimedia learning such as the *Cognitive Theory of Multimedia Learning* (CTML, Mayer, 2005) and the *Integrated Model of Text and Picture Comprehension* (ITPC, Schnotz, 2014; Schnotz & Bannert, 2003) assume dual coding and dual channel processing of information in sensory and working memory. Hence, these theories provide substantiated explanations for the benefits of certain types of MERs, such as illustrated text or lectures accompanied with pictures, which are presented to the audience on a wide screen (multimedia effect).

However, the basic definition of MERs also encompasses homogeneous combinations of representations, i.e., representations of the same type (e.g., Seufert, 2003). This is the case, for example, for multiple symbolic representations, which are the focus of the present research. In particular, within the field of STEM education (*s*cience, *t*echnology, *e*ngineering, and *m*athematics), specific symbolic representations apart from text, such as mathematical or chemical formulas and equations, are used and combined. The common theories on multimedia learning do not refer to the effects of combining multiple symbolic representations. In contrast, Ainsworth (2006) described external representations not only by their form of representational system but also by considering further dimensions of representations.

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According to her framework, MERs can be beneficial for learning even if they are not heterogeneous in the classical sense, if they support at least one of their specific advantageous functions.

The main aim of the present study was to investigate whether the benefits of MERs hold true for the combination of two symbolic representations (text vs. text + formula; formula vs. text + formula), and whether the strength of the effect differs from the common multimedia effect (text + formula vs. text + graphic). As study material, a set of propositional logic tasks – represented as text, formulas, or graphics – were pretested first and subsequently used in two experimental studies.

1.1. Theoretical models on learning with MERs

Multimedia learning means understanding concepts by using, for instance, more than one representational code. The most common way to integrate two representational codes is to present a written text together with a matching picture. Learning with such MERs has been proven to result in better learning outcomes than learning with the respective text alone (Butcher, 2014; Mayer, 2001), even when tested after a delay (Schweppe, Eitel, & Rummer, 2015). This phenomenon is called the *multimedia effect* (Mayer, 2009) and is one of the most well-known findings in learning with text and pictures (cf. Levie & Lentz, 1982; Vekiri, 2002 for reviews). It is regarded as a “benchmark finding” (Schweppe et al., 2015, p. 24) for every theory of multimedia learning.

The effectiveness of combining representations which differ for their representational code is usually explained based on assumptions regarding information processing and the construction of mental models. Two of the most prominent theories in this context are the CTML (Mayer, 2005; 2009), and the ITPC model (Schnotz & Bannert, 2003; Schnotz, 2005, 2014). The CTML is based on the multi-store memory model (Atkinson & Shiffrin, 1971), Paivio's dual coding theory (1986), and Baddeley's dual channel working memory model (1992). Based on Paivio (1986), the CTML assumes that verbal (words) and non-verbal information (pictures) are processed in two different cognitive subsystems, resulting in two specific mental representations. These separate mental representations are integrated into a coherent mental model when appropriate prior knowledge is retrieved from long-term memory. The multimedia effect is explained by the assumption that illustrated text appeals to both methods of information processing (verbal and pictorial), which results in more available and more sophisticated mental models compared to text alone. Mason, Tornatora, and Pluchino (2013), for example, used eye tracking to investigate specific cognitive subprocesses, such as selection and integration processes, which according to CTML underlie learning with illustrated text.

The ITPC model (Schnotz, 2005; 2014) is consistent with the CTML regarding its main assumptions. According to Schnotz, multimedia learning requires the learner to construct internal representations by making use of the external representations provided. The ITPC model describes the cognitive structures and processes which are involved when learners use multiple sensory modalities and distinctive forms of representation.

At an early stage of their theoretical work, Schnotz and Bannert (2003) focused on explaining the formation of internal representations dependent on the representational code of the provided information. The authors emphasized that all variants of representations can be classified either as descriptions or as depictions. Descriptions were characterized as texts and all other kinds of symbolic representations, such as mathematical expressions and formulas that consist of symbols. Depictions, however, are pictures and all other kinds of analogous representations that consist of icons.

According to the ITPC model, the multimedia effect can be explained not only for text and picture but also for all combinations including descriptive and depictive representations. Analogous to the CTML, the benefit of multimedia instruction can be attributed to dual coding, as information from different kinds of representations is assumed to be processed in two subsystems: the descriptive and the depictive branches. Each branch of information processing leads to a specific mental representation: a propositional representation (descriptional branch) and a mental model (depictional branch). Both internal representations and the two branches are constantly interacting and exchanging information. Thus, multimedia instruction fosters the construction of both kinds of mental representations, and therefore enhances both text and picture comprehension. However, Schnotz (2010) admitted that high prior knowledge can compensate for the lack of a second external representation.

Overall, both theories focus on heterogeneous combinations of MERs, characterizing them by the form of the involved external representations, their representational code, and their modality.

In her *DeFT* (*De*sign, *F*unctions, and *T*asks) framework, Ainsworth (2006) used the term *design* to refer to these characteristics of representations (and many others such as specificity, dimensionality, etc.), but also considered two other features of MERs – *tasks* and *functions* – in order to understand their effectiveness. The first feature refers to the fact that MERs set distinct cognitive tasks which the learner must perform when he or she interacts with the provided representations. Usually, this involves understanding each single representation (e.g., how the information is encoded) and how they relate to each other. According to Ainsworth (1999, 2006), multiple representations can be particularly beneficial if they fulfill specific functions during learning. Ainsworth differentiated between the *complementary functions*, *constraining functions,* and *constructing functions* of representations. These functions are not mutually exclusive, but one set of representations can fulfill multiple functions (Ainsworth, Wood, & O'Malley, 1998).

Representations are considered as complementary if they either contain complementary information but trigger the same cognitive processes or if they support complementary cognitive processes but provide the same information. Complementary representations can foster learning, as the different representations might be appropriate for different tasks or provoke different processing strategies. Moreover, learners who are provided with multiple representations can choose the one they generally prefer or are more used to working with, which is assumed to foster comprehension (Plass, Chun, Mayer, & Leutner, 1998).

Another possible function of MERs is that one representation can constrain the interpretation of the other. To support learners’ understanding of an unfamiliar or abstract information, a more familiar representation is provided. For example, well-known misconceptions of particular representations could be dispelled by adding a verbal explanation of the concept in everyday language as a second representation and, vice versa, an unfamiliar representation can constrain the interpretation of a familiar one if the latter is ambiguous and information can be concretized by a further, although unfamiliar, representation.

The third function of MERs relates to their ability to foster the construction of a deeper understanding of the concepts being taught. Ainsworth (1999) proposed that MERs could be used to promote abstraction as one example of their constructing function, which can mean either subtracting a concept to its essentials, identifying underlying patterns of a concept, or creating a completely new higher-level mental entity. According to Ainsworth (1999), abstraction can only result if the learner works actively with the provided representations by translating one into the other or constructing references across them. MERs can also support generalization as another example of the construction of deeper understanding. Thus, if a learner understands a concept which is represented in a certain way, he or she can learn to generalize this knowledge by learning how the same concept is represented in another fashion.

Ainsworth (1999) further claimed that MERs can be used to teach explicitly how to relate different representations to one another. In contrast to the generalization function, both representations are new to the learner and translation processes are performed bi-directionally.

To predict performance, it is crucial to know which functions are