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The impact of peer solution quality on peer-feedback provision on geometry proofs: Evidence from eye-movement analysis

Maryam Alqassab[a,*,1], Jan-Willem Strijbos[a,b], Stefan Ufer[c]

[a] Department of Psychology, LMU Munich, Germany
[b] Faculty of Behavioural and Social Sciences, University of Groningen, the Netherlands
[c] Mathematics Institute, LMU Munich, Germany

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ABSTRACT

Providing feedback on peer solutions to geometry proofs can support preservice mathematics teachers' assessment skills of such complex tasks. However, the quality of peer solutions may influence cognitive processing during peer-feedback provision, learning from providing peer-feedback, and peer-feedback content. To investigate this effect, we recorded the eye-movements of fifty-three preservice mathematics teachers while providing feedback on a near-correct or an erroneous peer solution to a geometry proof, and we measured their proof comprehension and peer-feedback content. Results show that the absence of errors earlier in the peer solution facilitated reliance on a figure-based approach, whereas encountering errors earlier in the peer solution was associated with more focus on the text of the proof. Students who provided peer-feedback on the near-correct peer solution had better comprehension of the proof, and they provided more accurate peer-feedback. Errors in peer solutions thus appear to hinder positive peer-feedback outcomes.

1. Introduction

Proof is central to mathematics instruction in schools (National Council of Teachers of Mathematics, 2000). Yet, there is a consensus in the literature that proof is challenging for high-school and university students (for a review see Harel & Sowder, 2007). In this context, proof is understood as a more or less formal, deductive argumentation establishing the validity of a mathematical statement, based on definitions and proven theorems from a framework theory (Stylianides, 2007). Students' weakness in proof is attributed to several factors including passive learning (e.g., observing a teacher, learning from textbooks). Research has shown that making proof instruction more active (e.g., through self-explanation) can improve students' understanding of proofs (i.e., proof comprehension; Hodds, Alcock, & Inglis, 2014).

Peer-feedback has the potential to stimulate active learning of proofs. Providing peer-feedback involves judging the correctness of a peer solution (e.g., proof) and producing statements to support or explain these judgements. Unlike other active learning techniques (e.g., self-explanation), peer-feedback on proofs involves judging the correctness of a proof constructed by another source (i.e., the peer). This proof validation activity is essential to proof instruction because it can help students to develop the skills to assess their own learning while constructing proofs (Selden & Selden, 2015a). However, students seldom encounter proof validation activities in mathematics classes as they are mainly exposed to correct proofs during instruction (Zerr & Zerr, 2011).

Peer-feedback is increasingly used in teacher-training courses, including mathematics education (e.g., Lavy & Shriki, 2014; Sluijsmans, Brand-Gruwel, Van Merriënboer, & Bastiaens, 2003), because it supports students' learning (Cho & Cho, 2011) and their assessment skills (Sluijsmans et al., 2003); the latter is a skill that every preservice teacher needs to develop. Preservice mathematics teachers particularly need to be able to assess proofs because most school mathematics curricula typically include them (Selden & Selden, 2015a). However, peer-feedback provision on proofs is likely to be challenging for preservice mathematics teachers. Studies on proofs showed that when undergraduate students are asked to validate proofs of different levels of correctness they could not reliably differentiate between correct and erroneous proofs (e.g., Inglis & Alcock, 2012; Selden & Selden, 2003), and that erroneous proofs are more challenging to the students to validate (Inglis & Alcock, 2012; Zerr & Zerr, 2011). Nevertheless, students' accuracy in proof validation seems to depend on the type of error in the proof (Sommerhoff, Ufer, & Kollar, 2016).

These findings are in line with peer-feedback research revealing that

* Corresponding author. LMU Munich, Department of Psychology, Leopoldstr. 44, 80802, Munich, Germany.
E-mail addresses: m.j.n.alqassab@gmail.com (M. Alqassab), j.w.strijbos@rug.nl (J.-W. Strijbos), ufer@math.lmu.de (S. Ufer).
[1] Permanent address: House 1944, Road 644, Sanabis 406, Kingdom of Bahrain.

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peer-feedback provided by a student is shaped by the quality of the peer solution (e.g., Patchan & Schunn, 2015) which can be reflected in the type of error in the proof. Peer-feedback studies have been focusing on improving the content of peer-feedback provided by students through instructional scaffolds (e.g., Alqassab, Strijbos, & Ufer, 2018; Gielen, Peeters, Dochy, Onghena, & Struyven, 2010). Yet, this approach did not sufficiently work for peer-feedback providers with low domain knowledge (Alqassab et al., 2018). Such research needs to be informed by empirical studies investigating how peer-feedback providers deal with the peer solution during the peer-feedback provision process, thereby producing (in)accurate peer-feedback or learning outcomes (e.g., comprehension of the proof). Specifically, we need to simultaneously investigate the process of composing the peer-feedback message and its outcomes to better understand this complex activity. Accordingly, there is a need to explore how the quality of peer solution influences process measures such as cognitive processing of the peer solution during peer-feedback provision as well as outcome measures (peer-feedback content and proof comprehension) in order to deliver more efficient instructional support for preservice mathematics teachers during this activity.

Eye-tracking is a useful tool to infer cognitive processes during assessment-related activities based on the assumption that what is being attended to is also cognitively processed (the eye-mind assumption; Just & Carpenter, 1976). Previous eye-tracking studies that investigated proof validation (e.g., Inglis & Alcock, 2012) or processing of peer-feedback by recipients (e.g., Bolzer, Strijbos, & Fischer, 2015) provided insights into the elements of proofs heeded during proof validation and how cognitive processing of received peer-feedback is related to revision. However, no study—to our knowledge—has investigated cognitive processing during peer-feedback provision on peer solutions to proofs despite the need for process measures underlying the outcomes (i.e., peer-feedback content and proof comprehension) of this challenging activity.

A specific type of proofs that is often used as initial context in proof instruction is geometry proofs. Their usefulness is attributed to the figure component that allows students to explore mathematical concepts visually and more easily by linking them to physical objects in the real world (Schoenfeld, 1986) and making inferences from the figure in geometry proofs is assumed to be easier than making inferences from statements (Larkin & Simon, 1987). However, geometry proofs are still widely ignored in research on proof in mathematics education despite (a) the usefulness of figures for learning as exemplified in multimedia learning research in different domains with the help of eye-tracking methodology (for a review see Eitel & Scheiter, 2015) and (b) the emerging interest in implementing peer-feedback activities with preservice mathematics teachers on geometry proofs (e.g., Lavy & Shriki, 2014). Yet, empirical studies investigating how preservice mathematics teachers utilize the figure of the geometry proof during peer-feedback provision are still limited.

1.1. Geometry proof based on mental models

Dealing with geometry proofs requires multiple skills including deductive reasoning (Schoenfeld, 1986). Several psychological theories about deductive reasoning can be applied to proof construction (for reviews see Bara, Bucciarelli, & Lombardo, 2001; Stylianides & Stylianides, 2008). We use the Mental Model Theory (Johnson-Laird, Byrne, & Schaeken, 1992) because it has previously been extended to research on geometry proofs (Ufer, Heinze, & Reiss, 2009), and is frequently utilized in research on learning with text and figure (see Schnotz, 2002).

Mental models are internal representations of premises or perceptual information in the external world that can be in the form of pictures, strings or symbols (Johnson-Laird et al., 1992). The Mental Model Theory postulates that deductive reasoning involves three phases. First, a mental model is created based on perceived verbal or perceptual premises. Second, a parsimonious conclusion is formulated based on information available within the mental model but not provided directly by the premises. Third, the conclusion is validated by checking that no alternative models of the premises violate this conclusion. If an alternative model of the premises refuting the conclusion is found the current conclusion is rejected, and phase two is repeated again (Johnson-Laird et al., 1992).

Ufer et al. (2009) extended the Mental Model Theory to explain reasoning processes underlying geometric proof construction. Their framework acknowledges that geometry proof tasks are often accompanied by a geometric figure. Mental models in this framework are not restricted to a specific geometric figure, but also comprises conceptual properties that define the geometric configuration (i.e., figural concept; see Fischbein, 1993). Hence, generating a mental model during the first phase of deductive reasoning requires the integration of two types of information (i.e., premises): (a) verbal information (i.e., problem text), and (b) visual information (i.e., figure). Students then generate intermediate conclusions in the second phase based on their mental model, which are then validated in the third phase by either trying to exclude contradicting alternative mental models, or by referring to a theorem that excludes the existence of such alternative models. Reading a geometry proof (attempt), thus, can be described by two different approaches: a text-based approach, that focuses on the different statements in the text and their mutual relations, or a figure-based approach, that focuses on what the statements given in the text mean in terms of the geometric configuration. The next section elaborates on how these two approaches can be employed during peer-feedback provision on geometry proofs.

1.2. Employing geometry proof mental models during peer-feedback provision

In the context of proofs, providing peer-feedback entails reading a proof attempt by a peer, reflecting on it, and judging its correctness (i.e., validation; Selden & Selden, 2015b). This act requires an involvement in all three phases of deductive reasoning described by the Mental Model Theory. In particular, the peer-feedback provider needs to construct a mental model (using information from the text and the figure) based on which s/he judges the correctness of the peer solution to produce peer-feedback.

Evidence from multimedia studies shows that a figure can support learning from text. For example, Eitel, Scheiter, Schüler, Nyström, and Holmqvist (2013) revealed that a figure acts as a mental scaffold that facilitates text-comprehension. Another study showed that the presence of pictures in items of a science test stimulated more efficient item-reading and better performance (Lindner, Eitel, Strobel, & Köller, 2017). Accordingly, we propose that adopting a figure-based mental model while providing peer-feedback on geometry proofs can facilitate feedback provision. Nevertheless, it is unclear under which conditions a figure-based approach is likely to be adopted. Unlike proof construction, in peer-feedback provision on geometry proofs the peer-feedback provider is presented with an already-constructed proof by a peer, thus the construction of the mental model is likely to be influenced by the quality of the written peer solution.

1.2.1. The role of peer solution quality in constructing mental models

During peer-feedback provision, the text of the peer solution represents the main body of the geometry proof. Hence, it is likely that the peer-feedback provider focuses on the text and inspects the figure in relation to the text. However, the quality of the peer solution might influence whether the peer-feedback provider adopts a figure-based or a text-based approach and to what extent s/he integrates both components of the peer solution to the geometry proof.

Eye-tracking studies suggest that when learning with text and figure (presented simultaneously), students focus mainly on the text and the processing of the figure is guided by the information available in the text (e.g., Eitel et al., 2013; Hegarty & Just, 1993; Stalbovs, Scheiter, &