Learning and Instruction 29 (2014) 78–94

Contents lists available at ScienceDirect

Learning and Instruction

journal homepage: www.elsevier.com/locate/learninstruc

Strategic counting: A novel assessment of place-value understanding

Winnie Wai Lan Chan [a,*], Terry K. Au [b], Joey Tang [b]

[a] Department of Psychological Studies, The Hong Kong Institute of Education, Hong Kong SAR, China
[b] Department of Psychology, University of Hong Kong, Hong Kong SAR, China

ARTICLE INFO

Article history:
Received 18 January 2013
Received in revised form
26 August 2013
Accepted 2 September 2013

Keywords:
Mathematical learning
Place-value understanding
Counting
Assessment

ABSTRACT

Children's counting strategies reflect how much they understand the place-value structure of numbers. In Study 1, a novel task, namely the strategic counting task, elicited strategies from kindergarteners and first graders that showed a trend of increasing place-value knowledge – from perceiving number as an undivided entity to seeing it as a collection of independent groups of powers of ten. In Study 2, first-graders' strategic counting task scores at the end of fall semester were better predictors of year-end mathematical achievement than the traditional place-value tasks. In Study 3, a five-item subset of strategic counting was the best among 15 various cognitive predictors of end of second-grade mathematical achievement. Growth curve modeling revealed that low-mathematics achievers at the end of second grade had been lagging behind their peers in strategic counting since early first grade. Implications for early support for children with difficulties in place-value knowledge are discussed.

1. Introduction

1.1. Place-value concept

Each digit in a multi-digit numeral carries a value that depends on its position – i.e., a place value. In the base-ten number system, each position represents a power of ten. In an integer, the right-most digit is in the 10^0 – or ones – place; the digit to its left is in the 10^1 – or tens – place; etc. For example, in the number 26, the digit 6 is in the ones place, so it carries a value of 6 (6×1), whereas the digit 2 is in the tens place, so it carries a value of 20 (2×10). The place-value concept seems important to mathematical learning and especially mastering arithmetic (Chan & Ho, 2010; Wearne & Hiebert, 1994).

Understanding the base-ten structure of numbers is crucial to mathematical problem-solving (Collet, 2003; Dehaene & Cohen, 1997; Fuson, Wearne, et al., 1997) and correlates with early mathematical achievement (Miura & Okamoto, 1989). Subpar place-value understanding predicts mathematical difficulties (Chan & Ho, 2010; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). In early grades, it predicts subsequent poor performance in addition and subtraction (Ho & Cheng, 1997) as well as in more complex arithmetic problems (Hiebert & Wearne, 1996). Place-value concept training improves arithmetic performance (Fuson, 1990; Fuson & Briars, 1990; Ho & Cheng, 1997; Jones, Thornton, & Putt, 1994). Hence, a good grasp of the place-value concept seems crucial in learning arithmetic.

Anyone who has difficulty with the place-value concept is apt to make errors in the comprehension and production of numbers, e.g., in counting as well as in reading and writing multi-digit numerals (McCloskey, 1992). Such errors tend to be syntactic, often leading to order-of-magnitude mistakes (e.g., "one thousand two hundred five" might be written as 10002005; Ginsburg, 1977). Procedures involving the base-ten structure of numbers, such as carrying over in addition and borrowing in subtraction, also become error prone (Fuson, 1990).

In much of the world, children receive formal instruction on the place-value concept in early elementary school. In Hong Kong, for instance, students learn about two-digit numbers in first grade, three- and four-digit numbers in second grade, and five-digit numbers in third grade. By the end of the third grade, students are expected to know how the place-value concept applies to all multi-digit numbers. Teachers usually teach this concept using base-ten manipulatives (e.g., base-ten-blocks) to illustrate the different places in a number concretely. They often assess place-value understanding by asking students to write down the number represented by the base-ten manipulatives or to identify the place value of a digit in a number.

1.2. Developmental stages of place-value understanding

By observing children's strategies for solving addition and subtraction problems, Fuson, Smith, and Lo Cicero (1997), Fuson,

* Corresponding author.
E-mail address: winniechan@ied.edu.hk (W.W.L. Chan).

0959-4752/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.learninstruc.2013.09.001

Wearne, et al. (1997) proposed the UDSSI model, which describes the developmental sequence of conceptual structures of two-digit numbers and offers insights into how the place-value concept develops. The model was named after five key conceptions: unitary multi-digit, decade and ones, sequence-tens and ones, separate-tens and ones, and integrated sequence-separate-tens. As new conceptions evolve, old conceptions may co-exist and be used in certain situations.

The unitary multi-digit conception extends children's conception of single-digit to multi-digit numbers: like a single-digit number, a multi-digit number represents an undivided quantity, and its individual digits are not meaningfully separable. For instance, the digits 2 and 6 in "26" in this conception are not associated with 20 and 6, or two groups of tens and six ones. The absence of a sense of partitioning differentiates this conception from the other four. This conception manifests itself in counting by ones without grouping (1, 2, 3, …, 26).

Developing a sense of partitioning depends on the structure of the number-naming system. Languages with separate decade words (e.g., twenty, thirty, forty in English) facilitate separating a quantity into decade and ones, resulting in the decade and ones conception. For instance, 26 is considered as twenty and six. With more experiences in counting by tens (e.g., ten, twenty, thirty, forty), children can develop the sequence-tens and ones conception, with the decade further separated into groups of tens. For instance, children count the quantity of 26 as 10, 20, 21, 22, …, 26 (Resnick, 1983).

In languages with a transparent base-ten number structure (e.g., "26" is rendered literally as "two-ten six" in Chinese), the groups of tens are clearly separate (i.e., two-tens in 26), fostering the separate-tens and ones conception (i.e., counting the groups of tens separately with a single-digit sequence). For instance, counting "26" as 1, 2 tens and 1, 2, 3, …, 6 ones. This conception, however, can be difficult to develop in number-naming systems where the groups of tens are not explicitly named (e.g., twenty-six in English). Finally, children may integrate these two latter conceptions (i.e., integrated sequence-separate-tens-and-ones conception) and readily shift back and forth between the two conceptions. For example, they can quickly tell that there are five tens in fifty, or seven tens make up seventy, without counting by tens.

The conceptions described in the UDSSI model – developing from whole-number quantity to base-ten partitioning – may signify growing place-value knowledge. This model also relates children's implicit knowledge about numbers with their counting behaviors, highlighting potential connections between basic numerical concepts and mathematical operations. These important theoretical implications remain untested empirically.

In the UDSSI model, each conceptual structure theoretically maps onto a counting strategy, whereas such mappings may not be one-to-one in reality. For example, the count-by-tens strategy may be used by children with the sequence-tens and ones conception (i.e., the theoretically associated conception) as well as those with the unitary multi-digit conception using rote learning. Even so, a strategy may be favored more by children with the theoretically associated conception than those without. Hence, we expected that the count-by-one strategy would be used by children with the unitary multi-digit conception more; the count-by-tens strategy would be used by those with the sequence-tens and ones conception more; the count-by-separate-tens-and-ones strategy would be used by those with the separate-tens and ones conception more.

Despite its usefulness, the UDSSI model needs elaboration. First, built upon informal observations of how children solved addition and subtraction problems, the UDSSI conceptual structures and developmental sequence remain untested empirically. Second, the model is silent on multi-digit numbers beyond two digits. Third, it was situated in the English number-naming system. Does it work for more regular systems in languages such as Chinese? For example, the numbers 11 and 12 are rendered respectively as "ten one" and "ten two" in Chinese – transparently mapped onto the base-ten structure – but obscurely as "eleven" and "twelve" in English. Previous studies have shown that a more regular number-naming system facilitates base-ten understanding (Fuson & Kwon, 1991; Miller & Stigler, 1987; Miura, Kim, Chang, & Okamoto, 1988; Miura, Okamoto, Kim, Steere, & Fayol, 1993; Miura et al., 1994). It remains to be seen whether the conceptions and developmental sequence described in the UDSSI model are language specific or universal.

1.3. Assessments of place-value concept

For assessment and research purposes, several place-value concept tasks have been developed. As it turns out, though, succeeding at these tasks is often possible without really understanding the concept.

A typical test – namely the number representation task – asks children to represent numbers with ten-blocks and unit-blocks (Miura et al., 1993; Naito & Miura, 2001; Saxton & Cakir, 2006; see Appendix A for other tests). Three kinds of constructions have been identified using this task (Miura et al., 1993; Naito & Miura, 2001; Ross, 1986): (1) one-to-one collection, where only the unit-blocks are used (e.g., 28 unit-blocks for the number 28), (2) canonical base-ten representation, where the numbers of ten-blocks and unit-blocks correspond to the base-ten structure (e.g., 2 ten-blocks and 8 unit-blocks for 28), (3) non-canonical representation, where the numbers of ten-blocks and unit-blocks do not correspond to the base-ten structure (e.g., 1 ten-block and 18 unit-blocks for 28).

One problem with this task is that many mathematics textbooks use illustrations of the canonical base-ten representation with ten-blocks and unit-blocks to teach the place-value concept. Children can learn by rote to represent the left digit of a two-digit number with ten-blocks and the right digit with unit-blocks. Moreover, a one-to-one collection does not necessarily imply a lack of the base-ten concept. Indeed, children who construct a one-to-one collection could construct a canonical base-ten representation when reminded that a ten-block is equal to ten unit-blocks (Miura & Okamoto, 1989). Therefore, this task can under- as well as over-estimate children's place-value understanding.

Another typical school assessment – intended to tap children's concept of base-ten partitioning – presents children with a picture of objects and asks them how many groups of ten can be formed. However, dividing a set of objects into groups of ten does not necessarily mean appreciating the base-ten structure of a multi-digit number (e.g., x units in the tens place stand for x groups of ten objects). Moreover, the task is limited by its assessing only a single aspect of the place-value concept – the concept of base-ten partitioning – without touching on other important aspects such as trading ten for one.

In most of other place-value tasks, children are given numbers and asked to do something with them (see Appendix A). But one task takes a different tack, which typically appears in mathematics exercises and textbooks. Children are shown sets of base-ten-blocks and asked to determine the numbers they represent. The sets are always arranged in canonical groupings corresponding to the base-ten structure (e.g., five ten-blocks and two unit-blocks for the number 52) Note that children may simply learn by rote to count the numbers of ten-blocks and unit-blocks and then string the two numbers together; here again, they may appear to understand the place-value concept more than they actually do (Ross, 1989).