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ScienceDirect

Nuclear Energy and Technology 3 (2017) 183–188

www.elsevier.com/locate/nucet

Quadrature formulas for integral equations of kinetics and digital reactimeters

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Available online 23 August 2017

Abstract

The aim of this work is to derive quadrature formulas for nuclear reactor kinetic equations in the form of Volterra integral equations of the second kind and reactimeter equations in the form of integral convolution, the kernel of which is a decay function of delayed neutron precursors (DNP) in the non-group form. The expediency of the transition to integral equations is caused by the unification of the direct (calculation of power dynamics) and the reverse (calculation of current reactivity) tasks of reactor kinetics. As a result, the solution is reduced to the calculation of the delayed neutrons integral (DNI). This eliminates the source of computational-experimental discrepancies in estimations of reactivity, which is due to the difference in computational algorithms of direct and inverse problems. The paper describes a general scheme for converting different transport equation approximations to describe the contribution of delayed neutrons by means of an integral convolution without using dynamic equations of the DNP concentration. This conversion reduces the model dimension, simplifies the software implementation, eliminates the stiffness problem of differential kinetic equations and provides the stability of calculations. The model dimension is preserved in the case of several fissile nuclides. The integral form of the equations makes it possible to use the experimental decay function in quadrature formulas, which can be identified in the operating conditions of a nuclear reactor and stored pointwise in a nongroup form without decomposition into the sum of exponentials. This eliminates the need to solve the non-linear problem of identifying group parameters of delayed neutrons and increases the adequacy of modeling. A series of quadrature formulas for the calculation of the DNI are obtained and the corresponding algorithms of a digital reactimeter and numerical simulation of the reactor kinetics are described.

Keywords: Dynamics of nuclear reactor; Point kinetics; Reactivity; Reactimeter; Integral equations; Quadrature formulas.

Introduction

In nuclear reactor physics much attention is paid to a comparison and correlation of calculated and experimental estimations of reactivity [1–4]. Such a comparison characterizes the accuracy and adequacy of neutron-physical calculations in the design, operation, and maintenance of nuclear safety of NPPs. However, as is known [5], differential equations for the description of the dynamics of delayed neutron precursors (DNP) are used in computational complexes, and the experimental estimation of reactivity is based on various versions of the inverse kinetics equation, in which it is easy to show that the contribution of DNPs is described by the convolution integral. Accordingly, various schemes for solving differential equations are used for calculations in the first case [6–8] and the simplest quadrature formulas in the second case [9]. The difference between mathematical models is one of the reasons for the discrepancy between the calculated and experimental results. To eliminate this factor, it is advisable to unify the computational models to ensure the identity of the solution schemes of the direct (power output calculation) and inverse (current reactivity calculation) tasks of the nuclear reactor neutron dynamics. Since the reactivity can be measured only by calculating the integral, the reactivity estimation in the computational modeling complexes must also be performed on the basis of integral equations using quadratures similar to those used in a digital reactimeter. More precisely,

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Peer-review under responsibility of National Research Nuclear University MEPhI (Moscow Engineering Physics Institute).

Russian text published: Izvestiya vuzov. Yadernaya Energetika (ISSN 0204-3327), 2017, n.2, pp. 93–105.

http://dx.doi.org/10.1016/j.nucet.2017.08.005

the equations must be transformed to integral form, and the subsequent discretization should be performed in the same way in both the direct and inverse problems using both traditional quadratures and other known approaches to solving integral equations [10,11].

The paper describes a general scheme for the rearrangement of various approximations of the transport equation to account for the contribution of delayed neutrons by means of the convolution integral. The proposed unification reduces the direct and inverse problems of kinetics to the calculation of the delayed neutron integral (DNI). A series of quadrature formulas for the calculation of the DNI are obtained and the corresponding design schemes for the realization of a digital reactimeter and numerical simulation of the kinetics of the reactor are described. The stability condition for computations is found.

Integro-differential and integral equations of neutron kinetics have long been used in modeling nuclear reactors [6–8,12–26]. The unification of the direct and inverse problems of nuclear reactor kinetics considered in this paper seems to provide a number of improvements in addition to the traditional approaches, namely:

- the model dimension decreases, only the observed quantities appear in the model;
- it becomes possible to use directly, as the kernel of the integral equation, the samples of the experimental DNP decay function;
- the transition to integral equations removes the problem of the stiffness of differential equations of nuclear reactor kinetics;
- it becomes possible to obtain interval estimations of reactivity on the basis of upper and lower integral sums [27];
- the standard metrological analysis schemes [28], based on convolution equations, become applicable for an analysis of reactimeter errors [29].

Unification of the direct and reverse problems of nuclear reactor dynamics

The integral representation of the source of delayed neutrons in the non-stationary transport equation is well known [17,30] and is written on the basis of the concept of the exponential character of decay of delayed neutron precursors in the following form (hereinafter all designations are standard):

$$Q^D(\mathbf{r},\mathbf{v},\tau)=\int_0^t \varphi(\mathbf{r},\mathbf{v},\tau)\sum_{j=1}^{J}\chi_j\beta_j\lambda_j e^{-\lambda_j(t-\tau)}d\tau + \sum_{j=1}^{J}\lambda_j c_j(\mathbf{r},0)\cdot e^{-\lambda_j t}. \quad (1)$$

Integral Summands (1) are solutions of the dynamic equations of DNP concentrations in the corresponding groups:

$$\lambda_j c_j(\mathbf{r},t)=-\frac{\partial c_j(\mathbf{r},t)}{\partial t}+\int \beta_j(\upsilon')\nu_j(\upsilon') \times \Sigma_{fj}(\mathbf{r},\mathbf{v}')\varphi(\mathbf{r},\mathbf{v}',t)d\mathbf{v}', \quad (2)$$

so that, in fact, these equations can be excluded from the computational schemes of the nonstationary transport equation since there is no special interest in the dynamics of DNP concentrations. We shall describe the elimination procedure for the non-stationary transport equation represented in the general form:

$$\frac{1}{\upsilon}\frac{\partial\varphi(\mathbf{r},\mathbf{v},t)}{\partial t}=R\varphi(\mathbf{r},\mathbf{v},t)-\sum_j\chi_j(\upsilon)\frac{\partial c_j(\mathbf{r},t)}{\partial t}. \quad (3)$$

Here, the change in DNP concentrations is taken into account by the second term, and the operator R combines all other processes and is interpreted as a reactivity operator. This equation is obtained by replacing the generation rate of delayed neutrons $\lambda_j c_j$, which appears in the traditional form of the transport equation, by the expression for $\lambda_j c_j$ from Eq. (2).

The initial concentrations of the precursors $c_j(\mathbf{r},0)$ are determined from Eq. (2) under the assumption of the reactor stationary state, i.e., when $\partial c_j/\partial t=0$. Therefore, it is convenient to introduce into Eq. (2) the variable $s_j \equiv \partial c_j/\partial t$, for which these equations take the form of the balance of accelerations of decay (generation) of the precursors:

$$\frac{\partial s_j(\mathbf{r},t)}{\partial t}=-\lambda_j s_j(\mathbf{r},t)+\int\beta_j(\upsilon')\nu_j(\upsilon') \times\Sigma_{fj}(\mathbf{r},\mathbf{v}')\psi(\mathbf{r},\mathbf{v}',t)d\mathbf{v}',$$

where $(\mathbf{r},\mathbf{v},t)\equiv\partial\boldsymbol{\rho},\mathbf{v},t)/\partial t$ is the rate of change of the neutron flux density. The initial condition here becomes zero, $s_j(\mathbf{r},0)\equiv 0$, causing the following kind of solution:

$$s_j(\mathbf{r},t)=\int_0^t e^{-\lambda_j(t-\tau)}\left[\int\beta_j(\upsilon')\nu_j(\upsilon')\Sigma_{fj}(\mathbf{r},\mathbf{v}')\psi(\mathbf{r},\mathbf{v}',\tau)d\mathbf{v}'\right]d\tau.$$

As a result, the problem of calculating the initial distribution of delayed neutron precursors is eliminated and the corresponding source of errors is eliminated.

Substituting $s_j\equiv\partial c_j/\partial t$ in Transfer Eq. (3) brings the latter into the form:

$$\frac{1}{\upsilon}\psi(\mathbf{r},\mathbf{v},t)=R\varphi(\mathbf{r},\mathbf{v},t)-I_{\text{çí}}(\mathbf{r},t)+Q \quad (4)$$

with the initial condition $(\mathbf{r},\mathbf{v},t)=(R(\mathbf{r},\mathbf{v},0)+Q)$. The contribution of delayed neutrons in Transport Eq. (4) is represented by the DNI:

$$I_{3\text{H}}(\mathbf{r},t)=\int_0^t\int W\left(\mathbf{r},\mathbf{v}',t-\tau\right)\psi\left(\mathbf{r},\mathbf{v}',\tau\right)d\mathbf{v}'d\tau, \quad (5)$$

the kernel of which is:

$$W(\mathbf{r},\mathbf{v}',t-\tau)=\sum_j\chi_j(\upsilon)e^{-\lambda_j(t-\tau)}\beta_j(\upsilon')\nu_j(\upsilon')\Sigma_{fj}(\mathbf{r},\mathbf{v}').$$

The described procedure for the change of variables is applicable to the elimination of dynamic equations of DNP concentrations in any approximations of the transport equation. In particular, it brings the system of differential point kinetic