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A study on pentagonal fuzzy number and its corresponding matrices

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A R T I C L E I N F O

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A B S T R A C T

In this article, the notion of pentagonal fuzzy number (PFN) is introduced in a generalized way. A few articles have been published based on this topic, but they have some ambiguities in defining this type of fuzzy number. Here, we proposed the logical definition in developing a pentagonal fuzzy number, along with its arithmetic operations. Based on PFN, the structure of pentagonal fuzzy matrices (PFMs) is studied, together with their basic properties. Some special type of PFMs and their algebraic natures (trace of PFM, adjoint of PFM, determinant of PFM, etc.) are discussed in this article. Finally, the notion of nilpotent PFM, comparable PFM, and constant PFMs, with their many properties, are highlighted in this article.

1. Introduction

Decision making problems in the real world are very often uncertain or vague in most cases. Fuzzy numbers are used in various fields, namely, fuzzy process modelling, control theory, decision making, expert system reasoning and so forth. Previous authors' studies on fuzzy numbers highlighted the arithmetic and algebraic structure based on triangular fuzzy numbers and trapezoidal fuzzy numbers. Fuzzy systems, including fuzzy set theory (Zadeh, 1965) and fuzzy logic, have a variety of successful applications. Fuzzy set theoretic approaches have been applied to various areas, from fuzzy topological spaces to medicine and so on. However, it is easy to handle the matrix formulation to study the various mathematical models. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology, we introduced the concept of pentagonal fuzzy number (PFN) and corresponding pentagonal fuzzy matrices (PFMs). Several authors have presented results of the properties of a determinant, adjoint of fuzzy matrices, and convergence of the power sequence of fuzzy matrices. A brief review on fuzzy matrices is given below.

The concept of fuzzy matrices was introduced for the first time by Thomason (Thomason, 1977) in the article entitled convergence of power of fuzzy matrix; later, Hashimoto (Hashimoto, 1983a) studied the fuzzy transitive matrix. The theoretical development of the fuzzy matrix was influenced through an article on some properties of the determinant and adjoint of a square fuzzy matrix proposed by Ragab et al. (Ragab and Eman, 1994). Moreover, some important results of the determinant of a fuzzy matrix were proposed by Kim (Kim et al., 1989). Several authors studied the canonical form and generalized fuzzy matrix (Hashimoto, 1983b; Kim and Roush, 1980), application of fuzzy matrices in a system of linear fuzzy equations (Buckley, 1991, 2001), etc. Some of the interesting arithmetic works on fuzzy numbers can be found in (Bhowmik et al., 2008; Dubois and Prade, 1979; Dubois and Prade, 1980). Conversely, some other articles studied different types of fuzzy numbers, namely, L-R type fuzzy number, triangular fuzzy number, and trapezoidal fuzzy number (Bansal, 2010). Thereafter, these types of fuzzy numbers were applied as a mathematical tool in the various fields of applied mathematics. The notion of a triangular fuzzy matrix was proposed for the first time by Shyamal and Pal (Shayamal and Pal, 2007) and was made familiar through introducing some new operators on triangular fuzzy matrices (Shayamal and Pal, 2004). The progression of fuzzy numbers became so fruitful that it spread into intuitionistic fuzzy matrices (Adak et al., 2012a; Adak et al., 2012b; Bhowmik and Pal, 2012; Bhowmik and Pal, 2008; Mondal and Pal, 2014; Pal, 2001; Pradhan and Pal, 2014a; Pradhan and Pal, 2014b; Pradhan and Pal, 2012; Shayamal and Pal, 2002) and interval valued fuzzy set theory (Mondal and Pal, 2015; Pal and Khan, 2005; Shayamal and Pal, 2006).

In this article, we introduce the notion of pentagonal fuzzy number in a well-defined manner by generalizing some other types

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of fuzzy numbers and studied the basic arithmetic and algebraic properties of the pentagonal fuzzy number. In Section 2, several preliminaries regarding the fuzzy number are presented. In Section 3, fundamentals of the pentagonal fuzzy number are established. Based on the pentagonal fuzzy number, the concept of pentagonal fuzzy matrix (PFM) is presented in Section 4. Some works related to nilpotent PFMs, comparable PFMs, and constant PFMs are studied in the remaining sections.

2. Preliminaries

We first recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 1. ***Fuzzy set.*** *A fuzzy set is characterized by its membership function, taking values from the domain, space or universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as* $A=(x,\mu(x);x\in X)$. *Here,* $\mu_A:A\to[0,1]$ *is the grade of the membership function and* $\mu_A(x)$ *is the grade value of* $x\in X$ *in the fuzzy set A.*

Definition 2. ***Normal fuzzy set.*** *A fuzzy set A is called normal if there exists an element* $x\in X$ *whose membership value is one, i.e.,* $\mu_A(x)=1$.

Definition 3. ***Fuzzy number.*** *A fuzzy number A is a subset of real line R, with the membership function* μ_A *satisfying the following properties:*

(i) $\mu_A(x)$ is piecewise continuous in its domain.
(ii) A is normal, i.e., there is a $x_0\in A$ such that $\mu_A(x_0)=1$.
(iii) A is convex, i.e., $\mu_A(\lambda x_1+(1-\lambda)x_2)\geq min(\mu_A(x_1),\mu_A(x_2))$. $\forall$ x_1,x_2 in X.

Due to wide applications of the fuzzy number, two types of fuzzy number, namely, triangular fuzzy number and trapezoidal fuzzy number, are introduced in the field of fuzzy algebra.

Definition 4. ***Triangular fuzzy number.*** *A fuzzy number* $A=(a,b,c)$ *is said to be a triangular fuzzy number if it has the following membership function*

Thus, the triplet (a,b,c) forms a triangular fuzzy number under this membership function. Graphically, its membership function looks like a triangle, which is depicted in Fig. 1.

Definition 5. ***Trapezoidal fuzzy number.*** *A fuzzy number* $A=(a,b,c,d)$ *is called a trapezoidal fuzzy number if it possesses the following membership function*

Graphically, the trapezoidal fuzzy number has a trapezoidal shape with four vertices (a,b,c,d), as depicted in Fig. 2.

However, real-life problems are sometimes concerned with more than four parameters. To resolve those problems, we propose another concept of the fuzzy number, called pentagonal fuzzy number (PFN). We discuss PFN in the next section.

$$\mu_A(x)=\begin{cases}0, & x\leq a\\ \dfrac{x-a}{b-a}, & a<x\leq b\\ 1, & x=b\\ \dfrac{c-x}{c-b}, & b<x\leq c\\ 0, & x\geq c.\end{cases}$$

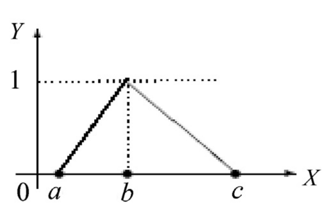

Fig. 1. Triangular fuzzy number.

$$\mu_A(x)=\begin{cases}0, & x\leq a\\ \dfrac{x-a}{b-a}, & a<x\leq b\\ 1, & b\leq x\leq c\\ \dfrac{c-x}{d-c}, & c<x\leq d\\ 0, & x\geq d\end{cases}$$

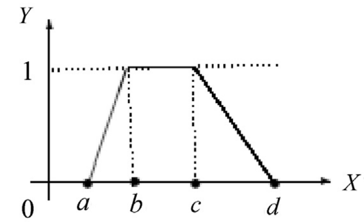

Fig. 2. Trapezoidal fuzzy number.

3. Pentagonal fuzzy number

Due to error in measuring technique, instrumental faultiness, etc., some data in our observation cannot be precisely or accurately determined. Let us consider that we measure the weather temperature and humidity simultaneously. The temperature is approximately 35°C with normal humidity, i.e., the temperature is not perfect either more or less than 35°C, which affects normal humidity in the atmosphere. Thus, variation in temperature also affects the percentage of humidity. This phenomenon happens in general. This concept of variation leads to a new type of fuzzy number called the pentagonal fuzzy number (PFN). Generally, a pentagonal fuzzy number is a 5-tuple subset of a real number R having five parameters.

A pentagonal fuzzy number A is denoted as $A=(a_1,a_2,a_3,a_4,a_5)$, where a_3 is the middle point and (a_1,a_2) and (a_4,a_5) are the left and right side points of a_3, respectively. Now, we construct the mathematical definition of a pentagonal fuzzy number.

Definition 6. ***Pentagonal fuzzy number.*** *A fuzzy number* $A=(a_1,a_2,a_3,a_4,a_5)$ *is called a pentagonal fuzzy number when the membership function has the form*

where the middle point a_3 has the grade of membership 1 and w_1,w_2 are the respective grades of points a_2,a_4. Note that every PFN is associated with two weights w_1 and w_2. To avoid confusion, we use the notation w_{iA} for $i=1{,}2$ to represent w_1 and w_2 as the weights of the PFN A.

3.1. Geometrical representation

From Fig. 3, it is clear that $\mu_A(x)$ has a piecewise continuous graph consisting of five points in its domain, forming a pentagonal shape. As chosen, the points in the domain have the ordering $a_1\leq a_2\leq a_3\leq a_4\leq a_5$; $a_1,a_2,a_3,a_4,a_5\in R$. We have to choose the value of the membership function at a_2,a_4 in such a way that $w_1\geq\frac{a_2-a_1}{a_3-a_1}$ and $w_2\geq\frac{a_1-a_5}{a_3-a_5}$. Otherwise, the convexity properties of the fuzzy number fail for the pentagonal fuzzy number.

Remark 1. *We define a pentagonal fuzzy number in a generalized way so that we can easily visualize two special fuzzy numbers, namely, triangular fuzzy number and trapezoidal fuzzy number, as follows:*

Case I When $w_1=w_2=0$, then the pentagonal fuzzy number is reduced to a triangular fuzzy number, i.e., $\tilde{A}=(a_1,a_2,a_3,a_4,a_5)\cong(a_2,a_3,a_4)$; in this case

$$\mu_A(x)=\begin{cases}0, & x\leq a_2\\ 1-\dfrac{a_2-x}{a_2-a_3}, & a_2<x\leq a_3\\ 1, & x=a_3\\ 1-\dfrac{a_4-x}{a_4-a_3}, & a_3<x\leq a_4\\ 0 & x\geq a_4\end{cases}$$

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