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Review article

Evaluation of the Triple Code Model of numerical processing—Reviewing past neuroimaging and clinical findings

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ABSTRACT

This review reconciles past findings on numerical processing with key assumptions of the most predominant model of arithmetic in the literature, the Triple Code Model (TCM). This is implemented by reporting diverse findings in the literature ranging from behavioral studies on basic arithmetic operations over neuroimaging studies on numerical processing to developmental studies concerned with arithmetic acquisition, with a special focus on developmental dyscalculia (DD). We evaluate whether these studies corroborate the model and discuss possible reasons for contradictory findings. A separate section is dedicated to the transfer of TCM to arithmetic development and to alternative accounts focusing on developmental questions of numerical processing. We conclude with recommendations for future directions of arithmetic research, raising questions that require answers in models of healthy as well as abnormal mathematical development.

What this paper adds: This review assesses the leading model in the field of arithmetic processing (Triple Code Model) by presenting knowledge from interdisciplinary research. It assesses the observed contradictory findings and integrates the resulting opposing viewpoints. The focus is on the development of arithmetic expertise as well as abnormal mathematical development. The original aspect of this article is that it points to a gap in research on these topics and provides possible solutions for future models.

1. Introduction

Over the past decades of research on arithmetic, TCM has become predominant in sketching the processes underlying arithmetic expertise and their interactions; however, the model is based on findings from educated adults and hence lacks specific explanations applicable to the development of arithmetic and to mathematical problems in DD (Kaufmann et al., 2013). In addition, few neuroimaging studies systematically tested TCM (according to Prado, Mutreja, & Booth, 2014), and the interaction of brain maturation and arithmetic education in healthy and dyscalculic children has not been clarified sufficiently (see Kaufmann, Wood, Rubinsten, & Henik, 2011 for a developmental calculation model based on a meta-analysis). The primary purpose of this review is to contrast those findings supporting the model with conflicting results from behavioral, clinical, neuroimaging, connectivity, and developmental studies conducted over the last 20 years. Moreover, a separate section is dedicated to developmental considerations and abnormalities evident in DD. The review aims at evaluating the current state of knowledge regarding TCM and provides possible reasons for contradictory findings. It finishes by outlining open questions as well as future perspectives focusing on applications that appear to be relevant in the diagnostics as well as interventions of DD.

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2. Scope of the present review

This review is an attempt to evaluate TCM from various perspectives in the fields of numerical processing and of basic arithmetic. We sought to compile findings from what we believe are the predominant questions in the literature, namely

(a) How are basic arithmetic operations solved?
(b) What are the roles of magnitude processing and arithmetic fact retrieval?
(c) What are the neural bases of the single arithmetic operations?
(d) How are brain regions involved in numerical processing connected?
(e) How do numerical processing and arithmetic develop?
(f) How can TCM be applied to healthy and abnormal arithmetic development?

These questions are addressed in the sections 'Empirical Evidence on TCM' (a to d) and 'Dyscalculia and TCM' (e and f). In addition, we elaborate on 'Possible reasons for discrepant results ", finishing with suggestions for 'Features of a comprehensive model of arithmetic development".

While we do believe that we have gathered the most relevant streams of research, providing opposing viewpoints for each, the present review is not systematic. We do not claim to provide an exhaustive overview of studies on arithmetic processing and TCM. Still, our review may initiate new ideas of thinking about numerical processing and lead to further elaborations on TCM when it comes to developmental aspects and abnormal development of arithmetic.

3. The Triple Code Model of number processing

3.1. Key aspects of TCM

Numerical processing is a complex skill involving several interrelated mechanisms such as understanding arithmetic principles or memorizing and retrieving arithmetic facts (Kaufmann et al., 2013). A clear-cut distinction has emerged between a system of calculation procedures based on quantity on the one hand and a second calculation system resting on memorized facts (Dehaene & Cohen, 1991). Incorporating this idea, TCM (Dehaene, 1992; extensions: e.g. Dehaene & Cohen, 1995; Dehaene & Cohen,1997) is a multiroute model of numerical processing postulating three functionally independent but interrelated codes (Dehaene, 1992). According to TCM, the semantic content of numbers (i.e. the meaning) is represented by an abstract magnitude module (M1, Dehaene & Cohen, 1995), whereas asemantic information is processed in two distinct modules, one for verbal numerical information and the other for written number words and Arabic digits (M2 and M3 respectively, Dehaene & Cohen, 1995). Together, M1 and M2/M3 represent separate but connected calculation systems (e.Dehaene, 2007).

Central to TCM is an assumed functional independence between notation types (Arabic digits vs. number words, see Knops, 2016) as well as the necessary transcoding pathways between the three codes (Dehaene & Cohen, 1995; Dehaene & Cohen, 1997). Thus, there is a direct route for converting visually presented input to verbal output and vice versa (M2; M3) independent of the semantic meaning (M1) of its quantity. Moreover, an indirect route processes the mental representation of quantity (Dehaene & Cohen, 1997). Depending on modality (visual vs. auditory see Knops, 2016) and notation, numerical content activates either M2 (i.e. verbal number words) or M3 (i.e. Arabic digits) or both, and may converge on the automatic activation of M1 which registers the associated numerical cardinality (Dehaene & Cohen, 1995; but see Wong & Szucs, 2013 contradicting automatic activation of quantity processing).

TCM predicts distinct paths of number processing depending on task complexity and the respective input, the mental operations carried out, and the resulting output. Consolidated arithmetic operations putatively become stored in long term memory and enable fast responses to simple arithmetic problems – potentially from an internal multiplication table (Krueger, Landgraf, van der Meer, Deshpande, & Hu, 2011). Thus, an efficient strategy to solve familiar multiplication problems especially of larger quantities is apparently to retrieve solutions from memory rather than by using calculation procedures (e.g. Zamarian, Ischebeck, & Delazer, 2009). As arithmetic facts are accessed linguistically, this process involves verbal phonological processing (M2) independent of the mental representation of magnitude (M1, see Moeller, Klein, Fischer, Nuerk, & Willmes, 2011).

Depending on task requirements, arithmetic problem solving can be exact or approximate (Klein, Nuerk, Wood, Knops, & Willmes, 2009), leading to distinct TCM pathways. In keeping with Dehaene and Cohen (1997), exact solutions to novel problems mandatorily involve calculation procedures and therefore activate verbal numbers (M2) or digits (M3). Exact calculation and fact retrieval are interrelated in that rehearsed arithmetic procedures (exact calculation) lead to strong associations between problem and solution in long term memory, allowing subsequent retrieval (Klein et al., 2016). As well, both share a common dependence on language (Rapin, 2016). By contrast, approximating a number range may be achieved by evaluating its cardinality using the so-called innate number sense (Dehaene & Cohen, 1997) or approximate number system (Cantlon, Platt, & Brannon, 2009), represented by M1 in TCM. Dehaene and Cohen (1995) assume that more complex arithmetic operations require "semantic elaboration" of the problem's magnitude, i.e. access to the cardinal meaning of the numbers. This may enable the recoding of the problem into simpler operations that are each solvable through fact retrievable. Therefore, fact retrieval and magnitude processing often go hand in hand in TCM.

To sum up, TCM is a multiroute model with distinct semantic and asemantic representations of numerical content accounting for double dissociations between arithmetic operations depending on input and output format.