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Representing contextual mathematical problems in descriptive or depictive form: Design of an instrument and validation of its uses

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ABSTRACT

The aim of this study is to contribute to the body of knowledge on the use of contextual mathematical problems. Word problems are a predominant genre in mathematics classrooms in assessing students' ability to solve problems from everyday life. Research on word problems, however, reveals a range of difficulties in their use in mathematics education. In our research we took an alternative approach: we designed image-rich numeracy problems as alternatives for word problems. A set of word problems was modified by systematically replacing the descriptive representation of the problem situation by a more depictive representation and an instrument was designed to measure the effect of this modification on students' performance. The instrument can measure the effect of this alternative approach in a randomized controlled trial. In order to use the instrument at scale, we made this instrument also usable as a diagnostic test for an upcoming nationwide examination on numeracy. In this article we explain and discuss the design of the instrument and the validation of its intended uses.

1. Introduction

In mathematics education there is an increasing focus on the usability of acquired mathematical knowledge and skills (Kilpatrick, 1996; De Lange, 1999; Toner, 2011), and hence there is a growing need for materials and tools to teach and assess the use of mathematical knowledge and skills in real-life situations. For decades it has been common practice to use word problems to teach and assess students' ability to solve quantitative problems in practical day-to-day situations (Verschaffel, Greer, & De Corte, 2000). However, the current practice of using word problems to assess students' ability to solve quantitative problems from everyday life also gives rise to serious concerns: The question is whether word problems are adequate for this purpose (Verschaffel et al., 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009).

According to Verschaffel, Depaepe, and Van Dooren (2014) word problems can be defined as "verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement" (p. 641). For this study we use word problems for which both the description of the problem situation as well as the actual problem statement are presented in words.

The reported difficulties with word problems are so persistent that in this study we investigated an alternative for word problems as means to evaluate students' ability to solve quantitative problems in practical day-to-day situations. In this alternative the descriptive representation of the problem situation, as is common in word problems, is replaced as much as possible by a depictive representation, which means using visual elements, mostly photographs, that were as close as possible to the real-life problem situation. To contrast them with word problems we call these problems image-rich numeracy problems. The choice for a more depictive representation of the problem situation is informed by research on difficulties with word problems, (Verschaffel et al., 2009), considerations on the sometimes problematic relation between language, context and sense-making in solving word problems (Sepeng, 2013), considerations on authenticity in mathematical problem solving (Palm, 2009; Verschaffel et al.,

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2000), and research on problem solving in cognitive psychology (Schnotz, 2002, 2005; Schnotz, Baadte, Müller, & Rasch, 2010). These research perspectives combined strongly suggests that using real-life images, such as photographs, to represent the problem situation has a better chance of keeping students in a problem solving mindset instead of falling back to an answer-getting mindset (Daro, 2013). Photographs are more easily associated with real-life situations, and arguably can feel more authentic for students, and therefore increase the chance that students continue using considerations of reality in the problem solving activities. Furthermore, it is likely that language and text comprehension difficulties are reduced by more depictive representations of the problem situation. The effects of changing the representation of the problem situation on students' performance are still under-researched and we decided a validated instrument was needed to measure these effects. In this article we described the design of such an instrument and the validation of its intended uses. The instrument was subsequently used in the Dutch context (Hoogland, De Koning, Bakker, Pepin, & Gravemeijer, submitted) and the English version of the instrument is now available under open access (Hoogland & De Koning, 2013).

2. Theoretical perspectives

2.1. Mathematics as usable knowledge

Over the past fifty years situations from real life have increasingly been used in the school mathematics classrooms. There are several developments in mathematics education that have pushed this trend forward. First, there is a plea by mathematics educators for a stronger emphasis in school mathematics on the ways in which mathematics is used in daily life (Kilpatrick, 1996; De Lange, 1999). Kilpatrick (1996) observes that "the curriculum had shifted (. . .) away from an emphasis on abstract structures towards efforts to include more realistic applications, with an emphasis on the ways in which mathematics is used in daily and professional life" (p. 7). Second and broadening the first development from applying to learning mathematics, there is an increasing use of examples from reality as integral part of an instruction theory for mathematics. For instance, in Realistic Mathematics Education (RME) (Freudenthal, 1973; Gravemeijer, 1994, 1999, 2004; Van den Heuvel-Panhuizen, 2000; Van den Heuvel-Panhuizen & Drijvers, 2014) contexts, models and representations play an important role in the educational process. The central idea in RME is that students should be supported in reinventing mathematics with the support of the teacher and the curriculum materials (e.g. textbook). The starting points of such reinvention processes should be experientially real for the students. Problems situated in every-day life contexts often fulfil this requirement. Hence in RME, situations from real life are not just used to prepare students for solving applied problems. The main function of real-life situations in RME is to offer a conceptual basis for reinventing the mathematics the students are to learn. In relation to this, Freudenthal's (1983) didactical phenomenology suggests to look for phenomena that—as he puts it—"beg to be organized" by the mathematical procedures, concepts or tools one wants the students to (re)invent. In addition to this, Treffers (1987) recommends a broad phenomenological exploration in order to incorporate various inroads to the mathematical procedures, concepts or tools under consideration. Third, in mathematics education research there is an increasing focus on problem-solving and modelling (Blum, Galbraith, Henn, & Niss, 2007; Burkhardt, 2006; Kaiser, Blomhøj, & Sriraman, 2006; Lesh and Zawojewski, 2007; Schoenfeld, 1992; Sriraman, Kaiser, & Blomhøj, 2006). Schoenfeld (2014) signals a reframing of what it means to understand mathematics:

> At the core of that reframing is the notion of mathematics as a sense making activity—that learning mathematics entails developing deep understandings of certain culturally and historically transmitted ideas, and employing those ideas in ways that reflect the perception of objects and relations, their mathematization, and the meaningful use of mathematical symbols in the service of solving problems. (p. 498)

The aforementioned developments concerning applicability, instruction, and problem solving and modelling can be seen as branches of a larger tree that represents the delicate relationship between reality and mathematics (education) and subsequently the way that reality is represented in classroom practice. As a consequence of these developments, in classroom practice we see a great variety of examples and problems from real life, which are used as tasks. The predominant form used for these tasks is the

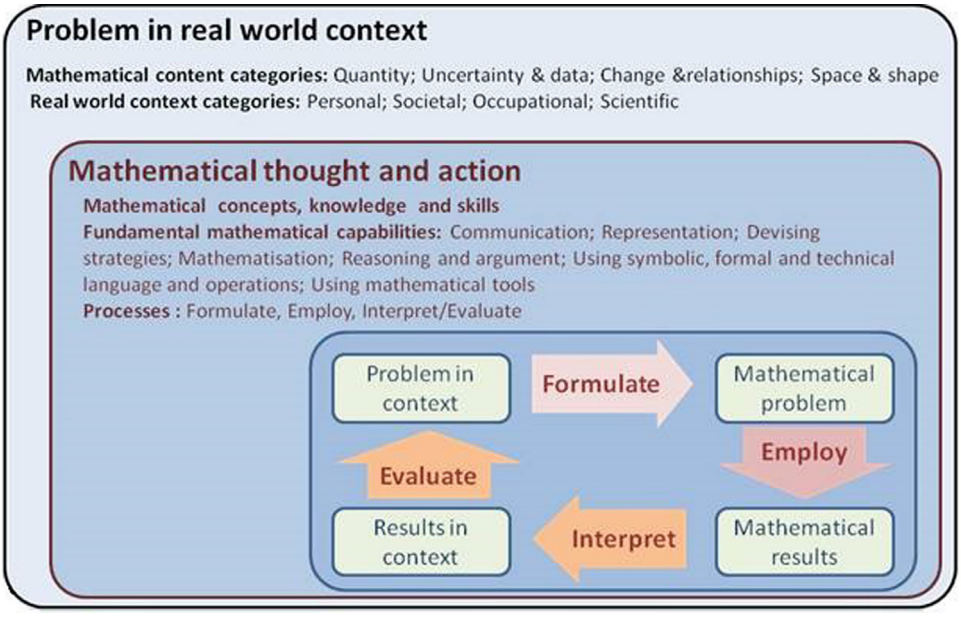

Fig. 1. A model of mathematical literacy in practice, according to OECD (PISA).