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Reconsidering the contribution of teacher knowledge to student learning: Linear or curvilinear effects?

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HIGHLIGHTS

- Prior works mainly examine linear effects of teacher knowledge on student learning.
- We explored both linear and curvilinear effects by focusing on fractions.
- A teacher survey and a student test aligned in content were developed and employed.
- Results supported a curvilinear effect of teacher knowledge on student learning.
- If replicated, these results have theoretical, methodological, and practical implications.

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ABSTRACT

Given mixed findings on the association between teacher knowledge and student learning, in this study, we revert to Shulman's framework to reconsider this association by exploring both linear and curvilinear effects. Toward this end, we analyzed data from a paper-and-pencil survey on teaching fractions completed by 373 teachers and a test aligned in content administered to students ($N = 1210$) of a subsample of these teachers. A two-level hierarchical linear model using scales developed by employing Item Response Theory suggested a curvilinear association between teacher knowledge and student learning for fifth graders. The theoretical, methodological, and practical implications of these results are discussed.

1. Introduction

During the last two decades significant advancements have been made in theorizing about the knowledge needed for teaching mathematics and in developing instruments to measure it (cf. Depaepe, Verschaffel, & Kelchtermans, 2013; Hill, Ball, Sleep, & Lewis, 2007). These advancements have led to accumulated empirical evidence supporting a positive moderate link between teacher knowledge and student learning (e.g., Baumert et al., 2010; Campbell et al., 2014; Hill, Rowan, & Ball, 2005; Rockoff, Jacob, Kane, & Staiger, 2011). Other studies (e.g., Cantrell & Kane, 2013; Kersting, Givvin, Thompson, Santagata, & Stigler, 2012; Shechtman, Roschelle, Haertel, & Knudsen, 2010), however, cast doubts about the contribution of teacher knowledge to student learning, thus underlining the importance of further investigating and understanding this link.

Interestingly, most of the existing studies exploring the association between teacher knowledge and student learning consider only linear effects,[1] assuming that students taught by more knowledgeable teachers are more likely to display larger learning gains. This assumption, however, raises a fundamental question: Could such effects also be curvilinear, implying that there is an optimal level of teacher knowledge before and after which teachers might be less effective in supporting student learning? This

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[1] Although strictly speaking "effects" refer to causal associations, in line with other studies exploring the contribution of teacher characteristics to student learning, our use of this term does not imply causation.

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possibility is reinforced by case studies reviewed in Section 2.2, showing mathematics teachers with either weak or strong knowledge struggling to support student learning.

This study aims to make a first step toward examining this possibility, by exploring both linear and curvilinear effects of the knowledge needed for teaching mathematics on student learning. To the extent that such curvilinear effects are identified, promising explanations for the mixed findings of prior studies exploring teacher knowledge and student learning could be advanced. Additionally, scholarly interest can focus on identifying and observing teachers with optimal levels of knowledge, attempting to understand how these teachers might deploy their knowledge to maximize student learning.

In what follows, the theoretical perspectives informing the current study are presented, followed by our research questions. Next, the methods pursued to address these questions are detailed. In the last two sections, we outline the study findings and, in discussing them, offer theoretical, methodological, and practical implications.

2. Theoretical perspectives

This section is organized into two parts. In the first, we review studies exploring linear effects of the knowledge for teaching mathematics on student learning. We do so to illustrate the main type of effects examined in prior related studies, but also to argue that the mixed findings identified in these works might be partly due to the type of the effects considered. Drawing on case studies and some evidence from large-scale studies, in the second part we make a case about the importance of also considering curvilinear effects. Due to space considerations, we do not consider the mediating variable of instructional quality; we, however, attend to this variable in considering the implications of the study findings and in offering ideas for future studies.

2.1. Exploring linear effects of teacher knowledge on student learning

During the past decade concerted efforts have accumulated significant empirical evidence linking teacher knowledge to student learning. This evidence derives from studies which, for simplicity reasons, can be grouped into two broad categories:[2] those adhering to Shulman's (1986, 1987) classification of teachers' content knowledge (CK) and pedagogical content knowledge (PCK) and those following Ball and colleagues' conceptualization of *Mathematical Knowledge for Teaching* (MKT, Ball, Thames, & Phelps, 2008).[3] In what follows, we review each category of studies, to illustrate that by and large scholars have examined only linear effects of teacher knowledge on student learning; when curvilinear effects were postulated, these were either not empirically tested or were tested in a rather unintentional way.

Focusing on secondary education, scholars under the *Cognitive Activation* (COACTIV) project (Baumert et al., 2010) examined the effect of two types of knowledge, CK and PCK, on student learning. Their exploration yielded a linear effect for PCK, according to which students taught by mathematics teachers who differed in their PCK by two standard deviations exhibited a difference of 0.46 standard deviations in their mean mathematics achievement by the end of the school year. A linear effect of teacher CK on student learning was also found; yet, CK had lower predictive validity for student progress compared to PCK. Interestingly, when the authors added a quadratic PCK term in their model, its effect was not significant.

Unlike the prior study that focused on Grades 9 and 10, a recent study examined the contribution of teachers' CK and PCK to student learning in Grades 4–8 (Campbell et al., 2014). Similar to the pattern just discussed, the contribution of teachers' PCK to student learning was more prevalent than that of CK; however, this pattern held only for middle grades. In particular, for each standard deviation increase in middle-grade teachers' PCK, student achievement scores increased per 0.22 standard deviations, an effect that remained significant even after controlling for several teacher (background) characteristics. Interestingly, no statistically significant effect of teacher PCK was found for upper-elementary grades, while CK turned out to be a significant linear contributor to student performance in both upper-elementary and middle grades. No curvilinear effects were, however, examined in this study for either type of knowledge.

Empirical evidence becomes even more inconclusive when considering additional studies also drawing on the CK and PCK distinction. For example, in Tchoshanov's (2011) work, pure mathematical knowledge was not associated with students' performance (Pearson's $r = .06$, $p = .54$); in contrast, one of the types of knowledge that reflected PCK—namely teacher knowledge of concepts and connections—had a low, yet significant linear correlation with student performance ($r = .26$, $p < .01$).

Drawing on the MKT conceptualization, a series of studies has also examined linear effects of teacher knowledge on student learning. Using data from first- and third-grade teachers, and largely focusing on common content knowledge (CCK) and specialized content knowledge (SCK), Hill and colleagues (Hill et al., 2005) found that teacher knowledge significantly predicted student gains in both grades. Specifically, a standard deviation difference in teachers' knowledge was found to be associated with gains of approximately 2.25 points in both first-and third-grade students' performance—approximately as large as those associated with the effect of students' socioeconomic background. Similarly, using an MKT survey and considering linear effects, Rockoff et al. (2011) found that students of teachers who performed one standard deviation higher on the MKT scale performed 0.025 standard deviations higher than their counterparts taught by average MKT teachers. However, teacher knowledge predicted less than 4% of the teacher-level variation, a point to which we return when discussing the findings of the present study.

Although the preceding two studies converge in suggesting that teacher knowledge matters for student learning, a series of other studies cast doubts regarding the contribution of MKT to student learning. For example, drawing on MKT, a set of studies conducted by Shechtman et al. (2010) examined the relationship between teachers' mathematical knowledge and student outcomes in three pre-algebra topics. Results suggested a significant relationship between teachers' MKT and student gains in only one of the three studies. In that study, teachers' MKT was a significant but modest predictor of seventh-grade student gains in cognitively demanding

[2] Other classifications are definitely possible. Our attempt here is neither to advance a classification scheme for the existing pertinent studies nor to provide a comprehensive review of the literature. Rather, we use these categories to document the type of effects largely examined in prior studies, regardless of the conceptualization pursued.

[3] Shulman's (1986) notion of CK refers to teachers' substantive (i.e., basic concepts and principles) and syntactic (i.e., how validity is established) knowledge of a discipline. PCK refers to knowledge of formulating the subject-matter to make it comprehensible to students and knowledge of students' subject-specific (mis) conceptions. Building on Shulman's conceptualization, Ball et al. (2008) proposed the MKT framework, according to which Shulman' subject-matter knowledge is tapped not only by a *common-content-knowledge(CCK)* domain, but also by a content-knowledge domain that is thought to be unique for the work of teaching (i.e., *specialized content knowledge*, *SCK)*. PCK is also decomposed into *knowledge of content and teaching (KCT)*, which intertwines knowledge of teaching with knowledge of mathematics, and *knowledge of content and students (KCS)*, which forms a combination of knowing about students and knowing about the mathematics.