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Dynamics in matching and coalition formation games with structural constraints ☆

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ABSTRACT

Matching and coalition formation are fundamental aspects in the organization of many multi-agent systems. In large populations, the emergence of coalitions is often restricted by structural constraints under which agents can reorganize, e.g., local visibility or externality constraints among the agents. We study this aspect using a novel framework for dynamics with constraints within the popular domain of hedonic coalition formation games. We analyze the effects of structural constraints on the convergence of matching and coalition formation processes to stable states. Our main result are tight characterizations for the constraint structures based on which dynamic coalition formation can stabilize quickly. We show a variety of convergence results for matching and coalition formation games with different forms of locality and externality constraints. In particular, we propose and analyze a new model of graph-based visibility for coalition formation games and tightly characterize the graph structures that allow polynomial-time convergence – it can be achieved if and only if coalition formation is based on complete or star graphs.

1. Introduction

Stable matching and hedonic games Matching and coalition formation problems form the basis for a variety of assignment and allocation tasks encountered in computer science, operations research, and economics. In multi-agent systems, the formation of coalitions is a natural process to exploit synergies. In this domain, the model of hedonic games [7,18,25,26,59] represents a natural and versatile approach to analyze aspects of coalition formation processes in multi-agent systems.

Perhaps the most prominent domain of hedonic games are classes of stable matching. In the classic stable marriage problem, there is a set of men and a set of women. Each agent strives to find a partner from the other gender, and it has a preference list over possible partners. Given a matching (i.e., a set of mutually disjoint man–woman pairs), a *blocking pair* is a man–woman pair such that both partners strictly improve over their current match if the pair is formed. A matching without blocking pair is called a *stable matching*. Since its introduction by Gale and Shapley in 1962 [27], stable matching has been successfully applied to capture assignment problems in multi-agent systems in a large variety of applications, e.g., assignment of jobs to workers [5,41], organs to patients [55], and many others. In addition, stable marriage problems have

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been successfully used to study distributed resource allocation problems in networks [3,28,46]. As such, stable marriage occupies a central position in the analysis of decentralized coalition formation.

There are several natural generalizations of stable marriage that have been intensively studied. Stable roommates instances drop the assumption of two sides – there is simply a single set of n agents, and each agent can match to some (possibly arbitrary) subset of other agents [40]. More generally, hedonic games allow the formation of coalitions of three or more agents. Their main characteristics are that (1) each agent can be part of at most one coalition, and (2) the preference or payoff an agent receives for being in a coalition depends only on the agents in that coalition, but not on how the remaining agents are partitioned. A variety of stability concepts in hedonic games and their aspects have been studied in recent years (see for example [8,9,11,18]). We here concentrate on the natural approach of core-stability, where agents have cardinal preferences for the coalitions they are part of. A state is a set of non-overlapping coalitions, and a blocking coalition is a group of agents that could (all individually) improve by abandoning their current coalitions and forming a new one together. A state is core-stable if there is no blocking coalition.

Structural constraints and improvement dynamics In the standard model of hedonic games, improvement in terms of preferences is the only criterion for deviation to other coalitions. In contrast, there can be a variety of additional constraints that govern the coalition formation process. Our main contribution in this paper is the formulation of a general framework for hedonic coalition formation with additional constraints. We study the consequences of these constraints on the existence of stable states and their reachability by myopic improvement dynamics.

Within our general framework, we put a focus on constraints derived from two natural properties of multi-agent systems – local visibility and (positive) externalities. As a concrete application, consider the prominent special case of stable matching. In large matching markets, it is unrealistic to assume that agents have full information about all other agents and possible partners. Instead, agents are often only aware about a subset of the population. For example, consider agents looking for a partner to do a joint activity (such as, e.g., playing squash or chess). We would not expect to form all possible and profitable matching pairs instantaneously. Instead, a pair of actors first have to get to know (about) each other before they can start a joint project. For the task of finding a partner, one often relies on existing relationships from family or co-workers. As a stylized model, we can capture this idea by extending the standard model of stable matching using a network – possible matching pairs have to be connected in an underlying social network to be available for formation. More interestingly, by engaging in an activity with a new partner, we get to know their friends and partners and possibly discover new and better matches. When we incorporate this aspect, we arrive at what has been termed *locally stable matching* [5,19,23,31,37]. Here the agents of a (local) blocking pair must have a hop-distance of at most 2 in the underlying network of social contacts and existing matched pairs. A variation of this model can be used to study job markets, where we strive to match jobs to workers. It is known that a large portion of jobs are assigned based on social contacts to co-workers in the same firm. Here the social network is present only among workers, but firms can match to multiple workers [5].

More generally, instead of visibility it is also natural to imagine that social contacts have a positive externality (or "altruistic") effect on actors. Beyond anecdotal evidence, there are studies in experimental game theory that support this assumption [43,44]. In addition to social ties and friendship, such an altruistic or considerate behavior can also arise, e.g., due to the existence of formal contracts or business relations among agents. Recently, such effects have received increased interest in a variety of game-theoretic models, including potential games [21,33,34], matching [4], and hedonic coalition formation [49]. In this case, agents are assumed to consider the (negative) effects on others when forming a new match and thereby, e.g., "stealing" the current partner of one of their friends or family members. A frequently explored model in games with cardinal utility is to express the trade-off between individual improvement and externality using a numerical value $c_{u,v} \geq 0$ for each pair u, v of agents. The perceived utility of an agent u then is a weighted sum of his individual utility and the utilities of all other agents v (weighted with $c_{u,v}$). Here the agents of a (friendship) blocking pair must have improved perceived utilities.

In this paper, our goal is to shed light on the properties of a variety of extensions of stable matching and hedonic coalition formation games, in which coalition formation is restricted by additional constraints. Such constraints can arise due to many aspects, for example, spatial closeness, previous collaborations, or social ties based on family, friendship, or co-worker relations. The general model we study here includes a variety of special cases that have found recent interest in the literature, such as socially and locally stable matchings [5,6,19,23,31,37] or friendship stable matching [4]. In addition, we also outline a natural model of *considerate stable matching*.

Our interest is in the properties of these games when the system is governed by dynamic and myopic coalitional deviations. Intuitively, such systems will eventually converge to a stable matching or core-stable state (if it exists). We study improvement dynamics, that is, the type of dynamics arising when we allow the iterative resolution of blocking coalitions until a stable state is reached. A blocking coalition is resolved by deleting all overlapping coalitions from the state and then adding the blocking coalition – possibly leaving some agents single. Thus, some agents improve in terms of their preference while others deteriorate. In consequence, such a sequence of improvement steps can take very long until it reaches a stable state, or it might even run into cycles. Of course, such behavior is undesirable, and our main interest is to identify and characterize conditions under which stable states can be reached (quickly).

A common aspect in all the example domains discussed above is that a blocking pair – in addition to being an improvement in terms of their preference for both incident agents – also has to fulfill additional graph-theoretic properties. A stable matching in these variants is a matching that has no blocking pair that satisfies such additional properties. Conse-