



Implicit, explicit and speculative knowledge

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ARTICLE INFO

Article history:

Received 15 August 2015

Received in revised form 18 November 2017

Accepted 21 November 2017

Available online 2 December 2017

Keywords:

Modal logic

Awareness

Bisimulation

Expressivity

Multi-agent systems

ABSTRACT

We compare different epistemic notions in the presence of awareness of propositional variables: the logic of implicit knowledge (in which explicit knowledge is definable), the logic of explicit knowledge, and the logic of speculative knowledge. Speculative knowledge is a novel epistemic notion that permits reasoning about unawareness. These logics are interpreted on epistemic awareness models: these are multi-agent Kripke structures for propositional awareness (in each state an agent may only be aware of formulas containing occurrences of a subset of all propositional variables). Different notions of bisimulation are suitable for these logics. We provide correspondence between bisimulation and modal equivalence on image-finite models for these logics. Expressivity and axiomatizations are investigated for models without restrictions, and for models with equivalence relations for all agents (modeling knowledge) and awareness introspection (agents know what they are aware of). We show that the logic of speculative knowledge is as expressive as the logic of explicit knowledge, and the logic of implicit knowledge is more expressive than the two other logics. We also present expressivity results for more restricted languages. We then provide and compare axiomatizations for the three logics; the axiomatizations for speculative knowledge are novel. We compare our results to those for awareness achieved in artificial intelligence, computer science, philosophy, and economics.

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1. Introduction

Given two structures for knowledge and awareness, when do they contain the same information? When does an agent whose knowledge or beliefs are modeled in these structures know exactly the same in both? Can an agent have knowledge about things that it is unaware of? In this paper, we provide answers to all three questions. We propose a notion of *awareness bisimulation* to answer the first question: structures contain the same information if they are awareness bisimilar. The answer to the second question is a corollary of that: they should be the same (awareness bisimilar) for anything that the agent is aware of. And by employing awareness bisimilarity we propose a notion of *speculative knowledge* with which we can reason about unawareness. This we can do while preserving the expressivity of explicit knowledge. We

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provide an exhaustive comparison of explicit, implicit, and speculative knowledge in terms of bisimulation characterizations, expressivity, and axiomatizations.

Our work is rooted in the tradition of epistemic logic [21] and in particular multi-agent epistemic logic [26,7]. An early paper extending knowledge with awareness is Levesque's *A Logic of Implicit and Explicit Belief* [24]. In [24], what would later be coined 'unawareness' in [6] is modeled as the value 'unknown' for a proposition, which means 'neither true nor false', such that apart from true and false a fourth value is 'both true and false'. Levesque's analysis of implicit and explicit belief then only uses three values, not the value 'true and false'. Such fourth-valued logics are suitable for reasoning about inconsistency in communicating databases. They have since then followed a different line of development [2,10,30], which we will recall when reviewing dynamics of awareness, later. Levesque permits belief (and thus some form of awareness) of tautologies containing unaware variables. This is not taken along in the subsequent [6], but remained a contested topic in the awareness literature, e.g., in [28, p. 269], where it is discussed whether one should always be aware of $1 = 1$.

In *Belief, Awareness, and Limited Reasoning* [6], Fagin and Halpern propose a framework for awareness and knowledge that can be seen as a further development of [24], and that obtained foundational status in the knowledge and awareness corpus of literature that since then evolved, also incorporating independent lines of research in economics, including [6,27,28,13,19,15]; see also the survey [33]. We then have a modality for implicit knowledge, interpreted as modal accessibility, a modality for awareness of formulas, and a modality for explicit knowledge, interpreted as implicit knowledge plus awareness of a formula.

Roughly, one can distinguish a more *syntactically* flavored approach, wherein an agent is aware of the formulas in a given set, and where this set is a parameter in a given state of information [6,13,35], from a more *semantically* flavored approach, wherein the agent is aware of all formulas only containing a subset (the aware variables) of the set of all propositional variables [6,27,28,19].

The reason to call the former 'syntactic' is that it allows, for example, an agent to be aware of $p \wedge q$ but not of $q \wedge p$, namely when the former is in the set of aware formulas but the latter is not. In particular, we can then have that the agent is aware of p and $p \rightarrow q$ but not of q , so that, even if the agent knows that p and knows that $p \rightarrow q$, she still does not know q because she is not aware of q . Thus, the rationality of the agent is bounded or limited. Other principally syntactic approaches include [20,12,16].

The 'semantic' approaches include those wherein the agent is aware of all formulas only using a subset of all propositional variables. The reason to call this 'semantic' might be its popularity among economists using partitional information structures to define awareness. These are logic-free, and thus syntax-free, approaches, so to speak: we can also identify a propositional variable with its denotation in a model, a semantic proposition. Within the restriction of the aware variables, agents may be fully rational. This should rather be called lack of conception than lack of awareness. The partitional information structures of [27,28] can be seen as the single-agent case of the complete lattice of spaces of [19], and are equivalent to the (single-agent) class of epistemic awareness models in which each agent knows what she is aware of [13]; the complete lattices of spaces are equivalent to the same (multi-agent) class of epistemic awareness models in [15]. Our proposal falls straight into the semantic corner.

In such multi-agent logics with implicit and explicit knowledge, implicit knowledge is a primitive notion needed to define explicit knowledge, and one should be careful when mixing the two notions. Consider the following example. 'Knowledge' means 'explicit knowledge'.

Example 1. We are both quite interested in the outcome of the presidential elections. A candidate has illegally employed immigrants (proposition p). We both know this.

(X) A scenario wherein *you are uncertain whether I know p* (i), is different from a scenario wherein you know that I know p (ii). And my knowledge about what you know about me may then also be different. This is standard multi-agent epistemic logic.

(Y) Let us now assume that *you are uncertain whether I am aware of p* . It may then be that I know that you are uncertain whether I am aware of p . It is not problematic for me to reason about what you think of me in such a case.

(Z) Let us instead assume that *you are unaware of p* . So now, it is no longer the case that we both know that p . You don't know p , as you are unaware of p . You then cannot distinguish the scenario wherein I know p from a scenario wherein I don't know p , or the scenario wherein I am aware of p from a scenario wherein I am unaware of p . (If I am unaware of p , it is irrelevant whether I implicitly know p (i), or not (ii). Explicit knowledge is all the agents have.) Such scenarios are indistinguishable given your current level of awareness. Consequently, what I know about your knowledge should also be the same in both scenarios, and in particular my knowledge about your knowledge about me.¹

We will see that we can distinguish the scenarios under Z in a logic with implicit and explicit knowledge and awareness, but not in a logic with only explicit knowledge and awareness. These logics have been well investigated, but not what their distinguishing power is on a class of structures. In modal logics, determining the difference between structures requires a notion of bisimilarity. We propose different notions including one called *awareness bisimilarity*.

With awareness bisimilarity at our disposal, we can also consider other epistemic notions. We define an epistemic operator called *speculative knowledge*, which not only varies over all the propositions the agent is aware of in the accessible

¹ In Fig. 2, later, model T represents scenario X.i and T' scenario X.ii, and in Fig. 3, T_{+p} represents Z.i and T'_{+p} represents Z.ii.

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