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Strategyproof matching with regional minimum and maximum quotas ☆

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ABSTRACT

This paper considers matching problems with individual/regional minimum/maximum quotas. Although such quotas are relevant in many real-world settings, there is a lack of strategyproof mechanisms that take such quotas into account. We first show that without any restrictions on the regional structure, checking the existence of a feasible matching that satisfies all quotas is NP-complete. Then, assuming that regions have a hierarchical structure (i.e., a tree), we show that checking the existence of a feasible matching can be done in time linear in the number of regions. We develop two strategyproof matching mechanisms based on the Deferred Acceptance mechanism (DA), which we call Priority List based Deferred Acceptance with Regional minimum and maximum Quotas (PLDA-RQ) and Round-robin Selection Deferred Acceptance with Regional minimum and maximum Quotas (RSDA-RQ). When regional quotas are imposed, a stable matching may no longer exist since fairness and nonwastefulness, which compose stability, are incompatible. We show that both mechanisms are fair. As a result, they are inevitably wasteful. We show that the two mechanisms satisfy different versions of nonwastefulness respectively; each is weaker than the original nonwastefulness. Moreover, we compare our mechanisms with an artificial cap mechanism via simulation experiments, which illustrate that they have a clear advantage in terms of nonwastefulness and student welfare.

1. Introduction

In this paper, we consider a two-sided matching problem. In a standard two-sided matching problem, there are two types of agents, where one type called a *student* is matched to another type called a *school*. Multiple students can be matched to

☆ This paper is based on the authors' conference publication [9]. The difference between this paper and [9] is as follows. In this paper, we use a strictly more general model, in which regional minimum and maximum quotas co-exist, while Goto et al. [9] consider only regional minimum quotas. They developed two mechanisms, one is fair but wasteful, while the other is nonwasteful but not fair. Both mechanisms cannot handle regional maximum quotas. In this paper, we concentrate on fair mechanisms and develop two fair mechanisms, one of which can be considered as an extension of the fair mechanism developed in [9].

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Table 1
DA-based mechanisms with regional quotas. $*$ indicates our contribution, † indicates a special case that can be covered by our contribution. (KK: Kamada and Kojima [18], BFIM: Biro et al. [2], FITUY: Fragiadakis et al. [7], GIKYY: Goto et al. [10].)

		Maximum quotas		
		Individual	Hierarchical regions	General regions
Minimum quotas	None	DA	KK/BFIM GIKYY	**NP-complete***
	Individual	FITUY	†	
	Hierarchical regions	†	**PLDA-RQ*** **RSDA-RQ***	
	General regions	**NP-complete***		

a single school, and a school has an individual *maximum* quota: the number of students assigned to a school cannot exceed a certain limit. In this paper, we use the terms "students" and "schools" to represent agents in our model, but our results can obviously be applied to other two-sided matching problems, such as matching medical residents to hospitals, cadets to military branches, workers to firms, and so on.

The theory of two-sided matching has been extensively developed.[1] However, many real-world matching markets are subject to more general distributional constraints. For example, school districts might need at least a certain number of students in each school to operate, as in college admissions in Hungary [2], i.e., *individual minimum quotas* must be satisfied. Also, in the early 2000s, the United States Military Academy solicited cadet preferences over assignments to various branches and imposed minimum and maximum quotas on the number of students who could be assigned to each branch [25, 26]. Diversity constraints at schools can also be considered as a minimum quota problem, where students are divided into several types according to their socioeconomic status, and school districts impose a minimum quota for each type of students at each school. Furthermore, minimum/maximum quotas can be imposed on a set of schools (region) rather than on an individual school. One motivating example of this model is a hospital–resident matching problem. Assume a policy maker requires that a certain number of residents be assigned to hospitals on an isolated island. She also hopes to avoid an excessive concentration of residents in metropolitan areas. However, she does not want to interfere with how these residents are assigned within the hospitals on the island or in the metropolitan area.

Table 1 summarizes the existing works related to regional minimum/maximum quotas. We assume that individual maximum quotas are requisite since virtually all existing works assume the existence of individual maximum quotas.[2] When considering regional quotas, we distinguish a special case where the regions have a hierarchical structure. When only individual maximum quotas are imposed, the standard Deferred Acceptance mechanism (DA) [8] is widely used because it is strategyproof and stable. Since stability is decomposed fairness and nonwastefulness, DA is fair and nonwasteful. Also, regional maximum quotas, in which regions have a hierarchical structure, are considered in [2,10,18]. Individual minimum quotas are considered in [7] and two DA-based strategyproof mechanisms are developed. However, as shown in Table 1, a number of interesting combinations remain unexplored, which we examine in this paper.

More specifically, we first analyze the complexity for checking whether a feasible matching (which satisfies all regional quotas) exists or not, when we put no restrictions on regions. We then prove that this problem is NP-complete. Since checking the existence of a feasible matching is intractable in general, we concentrate on a special case where regions have a hierarchical structure. As described in Table 1, we consider the most general case in this setting, where each region and each individual school can simultaneously impose both minimum and maximum quotas.

A hierarchical structure is ubiquitous in any company, university, or military organization. When such an organization as a company allocates human resources, it is natural to assume that the obtained matching must satisfy the feasibility constraints at various levels in the organization hierarchy; each division, department, or section, has its own minimum/maximum quotas. Such feasibility constraints can naturally be represented by regional quotas.

We develop two strategyproof mechanisms based on the DA, which we call *Priority List based DA with Regional minimum and maximum Quotas (PLDA-RQ)* and *Round-robin Selection Deferred Acceptance with Regional minimum and maximum Quotas (RSDA-RQ)*. These mechanisms are inspired by two existing mechanisms that can handle regional maximum quotas [10, 18]. We say a matching is *fair* if no school wants to replace a student matched to it with another student who prefers to be matched with the school. Also, we say a matching is *nonwasteful* if student s wishes to be assigned to school c but is not accepted, then there must be a plausible reason, i.e., moving s to c violates some maximum/minimum quotas. A matching which satisfies fairness and nonwastefulness is called *stable* matching.[3] When regional quotas are imposed, a stable matching may no longer exist, i.e., fairness and nonwastefulness are incompatible. PLDA-RQ and RSDA-RQ are fair, thus the matchings obtained by these mechanisms can be wasteful. We show that the two mechanisms satisfy different versions of nonwastefulness respectively; each is weaker than the original nonwastefulness. Moreover, we compare our mechanisms with an artificial cap mechanism (in which regional quotas are removed by imposing artificial caps so that the

[1] See Roth and Sotomayor [24] for a comprehensive survey of many results in this literature.

[2] If we set each individual quotas large enough, e.g., equal to the number of students, it becomes non-binding. Thus, we can represent a case without individual maximum quotas.

[3] Technically, stability further requires individual rationality. However, in our environment where no agent is unacceptable, individual rationality is trivially satisfied. Based on this observation, we regard a stable matching as a fair and nonwasteful matching throughout the paper.