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The complexity of manipulative attacks in nearly single-peaked electorates ☆

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ABSTRACT

Many electoral control and manipulation problems—which we will refer to in general as "manipulative actions" problems—are NP-hard in the general case. It has recently been noted that many of these problems fall into polynomial time if the electorate is single-peaked, i.e., is polarized along some axis/issue. However, real-world electorates are not truly single-peaked. There are usually some mavericks, and so real-world electorates tend merely to be nearly single-peaked. This paper studies the complexity of manipulative-action algorithms for elections over nearly single-peaked electorates. We do this for many notions of nearness and for a broad range of election systems. We provide instances where even one maverick jumps the manipulative-action complexity up to NP-hardness, but we also provide many instances where some number of mavericks can be tolerated without increasing the manipulative-action complexity.

1. Introduction

Elections are a model of collective decision-making so central in human and multiagent-systems contexts—ranging from planning to collaborative filtering to reducing web spam—that it is natural to want to get a handle on the computational difficulty of finding whether manipulative actions can obtain a given outcome (see the surveys [36,12]). A recent line of work [64,40,10] started by Walsh [64] has looked at the extent to which NP-hardness results for the complexity of manipulative actions (bribery, control, and manipulation) may evaporate when one focuses on electorates that are (unidimensional) single-peaked, a central social-science model of electoral behavior. That model basically views society as polarized along some (perhaps hidden) issue or axis.

However, real-world elections are unlikely to be perfectly single-peaked. Rather, they are merely very close to being single-peaked, a notion that was recently raised in a computational context by Conitzer [15] and Escoffier et al. [32]. There will almost always be a few mavericks, whose vote is based on some reason having nothing to do with the societal axis. For example, in recent US presidential primary and final elections, commentators discussed whether some voters might vote not based on the political positioning of the candidates but rather based on the candidates' religion or race. In this paper, we study whether the evaporation of complexity results that often holds for single-peaked electorates will also occur in nearly single-peaked electorates. We prove that often the answer is yes, and sometimes the answer is no.

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Among the contributions of our paper are the following.

1. A collection of votes is said to be single-peaked if all the votes respect—i.e., are consistent with, in a sense we will rigorously define later—some societal axis. We classify and quantify four natural notions of nearness to single-peakedness for elections. The four notions are the following: (a) we allow some number of "maverick" voters, whose preferences need not respect the societal axis, (b) all voters must be consistent with the societal order when we disregard each voter's top preferred candidate (that is, each voter may have "swooned" regarding his or her top choice), (c) every vote is within a small number of adjacent-candidate swaps of respecting the societal order, and (d) all the votes are consistent with some axis, but that axis may be one that is not the societal axis but rather is within a small number of adjacent-candidates swaps of the original axis. For each of these notions we show how, given the society's axis, to compute in polynomial time how much the voters diverge from single-peakedness according to that notion.[1]
2. We establish the complexity of coalitional weighted manipulation under 3-candidate scoring protocols for nearly single-peaked elections. As summarized in Table 1 on page 79, we show that even a very slight deviation from single-peakedness, under any of our four distance models, can raise the manipulation complexity from P, which is what holds in the (perfectly) single-peaked case, to being just as hard as in the case of unrestricted preferences. Moving beyond three candidates, we extend our study of manipulation to m-candidate veto, for each m. As reflected in Table 2 on page 79, Theorems 5.3 and 5.4 establish a tight relationship between m and the amount of divergence from single-peakedness that can be handled in polynomial time.
3. We show, in all but one of the cases that we study, that the complexity of control problems for plurality, k-approval, and approval elections that are nearly single-peaked is the same as the control complexity for elections where the voters are (perfectly) single-peaked. For the only case (control by deleting voters under t-approval elections and swoon-SP societies; see Section 4 for the definitions) in which we do not reach the same result as for the case of single-peaked elections (i.e., where we do not obtain a polynomial-time algorithm), we construct a good polynomial-time approximation algorithm. Our results on this are summarized in Table 3 on page 82.

This paper touches on control and manipulation, discusses various election systems and notions of nearness to single-peakedness, and gives both polynomial-time attack results and NP-hardness results. It thus is not surprising that the proofs vary broadly in their techniques and approaches; we have no single approach that covers this entire range of cases.

The paper is organized as follows. Section 2 discusses related literature. Section 3 provides basic definitions regarding elections and the complexity of manipulating them. Section 4 defines what we mean by societies being nearly single-peaked and provides basic results and observations regarding our notions, justifying various modeling choices; some discussion related to this also can be found in the Related Work section. Section 4 also discusses median voting. Section 5 discusses the issue of manipulation in nearly-single peaked societies. Section 6 studies the extent to which nearness to single-peakedness affects the complexity of control problems. Section 7 provides conclusions and discusses some open problems.

2. Related work

Although it has roots going even further back, the study of the computational complexity of control and manipulation actions was started by a series of papers of Bartholdi, Orlin, Tovey, and Trick around 1990 [4,2,5]. The complexity of bribery—which is mentioned only in passing in the present paper—was first studied by Faliszewski et al. [33]. For further references, history, context, discussions, and results regarding these three types of manipulative actions, see the surveys [34,36]. For example, it is known that there exist election systems that are resistant to many control attacks [29,26,30,35,49].

The four papers most related to the present one are the following. Walsh [64] insightfully raised the idea that general complexity results may change in single-peaked societies. His manipulation example, which regards so-called single transferable vote elections, actually provides a case where single-peakedness fails to lower manipulation complexity, but in a different context he did find a lowering of complexity for single-peakedness. Faliszewski et al. [40] and Brandt et al. [10] then broadly explored the effect of single-peakedness on manipulative actions. These three papers are all in the model of (perfect) single-peakedness. Conitzer [15], in the context of preference elicitation, raised and studied the issue of *nearly* single-peaked societies. It is important to mention that he explicitly mentions two contrasting types of approaches to modeling near-single-peakedness, namely, having at most a few voters diverge, potentially greatly, from single-peakedness (this is the model on which his work concentrates); and having all voters allowed to be (in some sense) close to single-peakedness. The present paper explores both nearness of the former type, as our "maverick"-based notions, and nearness of the latter type, as our "swoon"-based, "perception flip"-based, and "Dodgson"-based notions. Escoffier et al. [32] also discussed nearness to single-peakedness, and Faliszewski et al. [40] and Brandt et al. [10] raise as open issues whether shield-evaporation (complexity) results for single-peakedness will withstand near-single-peakedness, i.e., whether, in cases where a general-case NP-hardness result drops to P for single-peaked electorates, the complexity will stay in P even for nearly single-peaked electorates. Elkind et al. [25], among other issues, study algorithms for measuring society's nearness to single-peakedness

[1] For notion (b), which we will call "swoon-SP," the society either does or does not satisfy the condition, so for that notion we instead provide a polynomial-time decision algorithm.