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Towards fixed-parameter tractable algorithms for abstract argumentation ☆

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A R T I C L E I N F O

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A B S T R A C T

Abstract argumentation frameworks have received a lot of interest in recent years. Most computational problems in this area are intractable but several tractable fragments have been identified. In particular, Dunne showed that many problems can be solved in linear time for argumentation frameworks of bounded tree-width. However, these tractability results, which were obtained via Courcelle's Theorem, do not directly lead to efficient algorithms. The goal of this paper is to turn the theoretical tractability results into efficient algorithms and to explore the potential of directed notions of tree-width for defining larger tractable fragments. As a by-product, we will sharpen some known complexity results.

1. Introduction

Argumentation has evolved as an important field in AI with abstract argumentation frameworks (AFs, for short) as introduced by Dung [20] being its most popular formalization. Meanwhile, a wide range of semantics for AFs has been proposed (for an overview see [4]) and their complexity has been analyzed in depth. Most computational problems in this area are intractable (see e.g. [17,24,26]), but the importance of efficient algorithms for tractable fragments has been clearly recognized (see e.g. [18]). Such tractable fragments are, for instance, symmetric argumentation frameworks [12] or bipartite argumentation frameworks [22].

An interesting approach to dealing with intractable problems comes from parameterized complexity theory and is based on the following observation: Many hard problems become tractable if some problem parameter is bounded by a fixed constant. This property is referred to as *fixed-parameter tractability* (FPT). One important parameter of graphs is the tree-width, which measures the "tree-likeness" of a graph. Indeed, Dunne [22] showed that many problems in the area of argumentation can be solved in linear time for argumentation frameworks of bounded tree-width. This FPT result was shown via a seminal result by Courcelle [13]. However, as stated in [22], "rather than synthesizing methods indirectly from Courcelle's Theorem, one could attempt to develop practical *direct* methods". The primary goal of this paper is therefore to present new, direct algorithms for certain reasoning tasks in abstract argumentation.

Clearly, the quest for FPT results in argumentation should not stop at the tree-width, and further parameters have to be analyzed. This may of course also lead to negative results. For instance, considering as parameter the degree of an argument (i.e., the number of incoming and outgoing attacks), Dunne [22] showed that reasoning remains intractable, even if decision

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problems are given over AFs with at most two incoming and two outgoing attacks. A number of further parameters is however, still unexplored. Hence, the second major goal of this paper is to explore the potential of further parameters for identifying tractable fragments of argumentation. In particular, since AFs are directed graphs, it is natural to consider directed notions of width to obtain larger classes of tractable AFs. To this end, we investigate the effect of bounded cycle-rank [28] on reasoning in AFs. We show that reasoning remains intractable even if we only consider AFs of cycle-rank 2. Actually, many further directed notions of width exist in the literature. However, it has been recently shown [6,33,31] that problems which are hard for bounded cycle-rank remain hard when several other directed variants of the tree-width are bounded. A notable exception is the related notion of clique-width [14] which (in contrast to tree-width) can be directly extended to directed graphs. Moreover, meta-theorems for clique-width [15] show that Dunne's result on tractability with respect to bounded tree-width extend to AFs of bounded clique-width (for details, we refer to [27]).

Still, the main focus of this paper is on novel algorithms for decision problems defined over the so-called preferred semantics of AFs. Roughly speaking, the preferred extensions of an AF are maximal admissible sets of arguments, where admissible means that the selected arguments defend themselves against attacks. To be more precise, we present here algorithms for the following three decision problems.

- Credulous acceptance: deciding whether a given argument is contained in at least one preferred extension of a given AF.
- Skeptical acceptance: deciding whether a given argument is contained in all preferred extensions of a given AF.
- Ideal acceptance: deciding whether a given argument is contained in an admissible set which itself is a subset of each preferred extension of a given AF.

The problem of ideal acceptance is better known as ideal semantics [21]. To the best of our knowledge, FPT results for ideal semantics have not been established yet, thus the algorithm that we present in the paper provides such a result as a by-product (one could alternatively use Courcelle's meta-theorem to obtain that result). By its very nature, the running times of our novel algorithms will heavily depend on the tree-width of the given AF, but are linear in the size of the AF. Thus for AFs of small tree-width, these algorithms are expected to be preferable over standard algorithms from the literature (see e.g. [19,38]).

One reason why we have chosen the preferred semantics for our work here is that it is widely used. Moreover, admissibility and maximality are prototypical properties common in many other semantics, for instance complete and stable [20], stage [43], and semi-stable [10] semantics. Hence, we expect that the methods developed here can also be extended to other semantics.

1.1. Summary of results

- We first prove some negative results: we show that reasoning remains intractable in AFs of bounded cycle-rank [28]. As has been mentioned above, this negative result carries over to many other directed notions of width. We also show that the problem of skeptical acceptance is coNP-complete for AFs of cycle-rank 1.
- We develop a dynamic programming approach to characterize admissible sets of AFs. The time complexity of our algorithm is linear in the size of the AFs (as expected by Courcelle's Theorem) with a multiplicative constant that is *single* exponential in the tree-width (which is in great contrast to algorithms derived via Courcelle's Theorem). This algorithm can be directly used to decide the problem of credulous acceptance.
- This dynamic programming algorithm is then extended so as to cover also the preferred semantics, and thus to decide skeptical acceptance.
- We finally show how to further adapt this algorithm to decide ideal acceptance.

1.2. Structure of the paper

In Section 2, we recall some basic notions and results on AFs and discuss some width-measures for graphs. We then show in Section 3 some negative results for reasoning in AFs where some parameters of directed graphs are bounded. In Section 4.1, we first develop a dynamic programming approach for credulous acceptance in AFs of bounded tree-width. This algorithm is then extended to cover also preferred semantics in Section 4.2 and adapted to ideal acceptance in Section 4.3. Section 5 provides some final conclusions as well as pointers to related and future work.

2. Background

In this section, we first introduce argumentation frameworks and then some graph measures we want to investigate for such frameworks.

2.1. Argumentation frameworks

We start by introducing (abstract) argumentation frameworks [20], and then recall the preferred as well as the ideal semantics for such frameworks. Afterwards, we highlight some known complexity results for typical decision problems associated to such frameworks.